(11) 0093

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Civil Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: CEX5233/ CVX5533 Structural Analysis**
Academic Year	: 2017/18
Date	: 20th February 2019
Time	: 0930-1230hrs
Duration	**: 3 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Relevant charts are provided.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Use of Calculator is permitted.
9. Do not use Red colour pen.

QUESTION 1

(i) Explain the significance of **St. Venant's Principle** in structural analysis. (3 Marks)

(ii) A mathematical function "ϕ" has been introduced to describe a stress field of a very thick component which has the same boundary condition on any given cross section. The elastic modulus and Poisson's ratio of the material are 20GPa and 0.2, respectively. The function is given as below

$$\emptyset = x^4y + 4x^2y^3 - y^5$$

(a) Show that this mathematical function is a possible stress function. (3 Marks)

(b) Determine **all** stress components. (4 Marks)

(c) Determine strain components. (4 Marks)

(iii) Write short notes on "**Null Method**" and "**Out of Balance Method**" in experimental stress measurements using electrical resistance strain gauges. (*Note: Draw Wheatstone bridge circuit*) (6 Marks)

QUESTION 2

(i) Discuss the difference between elastic neutral axis and plastic neutral axis using clear sketches. (3 Marks)

(ii) Consider a three-span continuous beam as shown in Figure Q2. Two concentrated loads "WL" and "2WL" act on spans, AB and CD, respectively. Further, a uniformly distributed load "W" acts on span BC. Flexural rigidity of the beams is shown in the figure. The plastic moment capacity of beams is M_p.

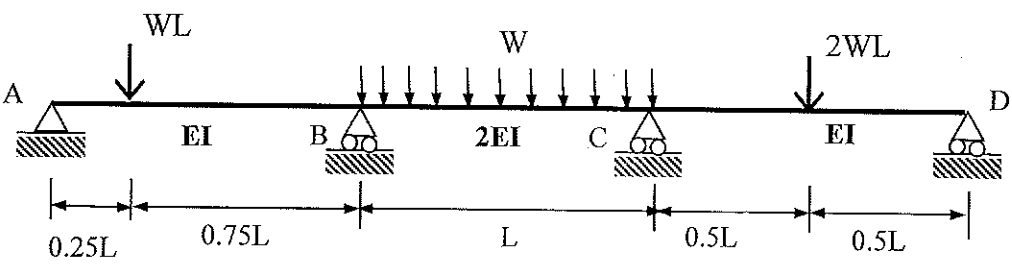


Figure Q2

a) Draw three different collapse mechanisms. (3 Marks)

b) Determine the load factors for above collapse mechanisms. (6 Marks)

c) Determine the most probable failure mechanism. (4 Marks)

d) Propose a suitable cross section for the beam and justify your selection. (4 Marks)

QUESTION 3

(i) Explain with clear sketches that stress **is not** a vector quantity. (3 Marks)

(ii) The stress tensor at a point of a particular stress field in Cartesian coordinate system is given below

$$\sigma_{ij} = \begin{bmatrix} 5 & 0 & 0 \\ \alpha & -6 & -12 \\ \beta & -12 & 1 \end{bmatrix}$$

a) Determine values of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, and draw the stress field. (3 Marks)

b) Determine the stress invariants. (4 Marks)

c) Determine the principal stresses. (4 Marks)

d) Determine the maximum shear stress. (3 Marks)

(iii) "The above stress field **is not** a plain stress field". Defend this statement with logical arguments (3 Marks)

QUESTION 4

(i) Explain the reason that statically indeterminate structures are more preferred over statically determinate structures in real world structures. (4 Marks)

(ii) Consider the continuous beam shown in Figure Q4. Flexural rigidity of members is equal to EI. Uniformly distributed load (*W*) is acting on member AB and a concentrated load (*WL*) is acting on the member CD at E.

a) Determine the degree of statical indeterminacy of the beam. (3 Marks)

b) Draw a released structure. (3 Marks)

c) Determine the flexibility matrix for the drawn released structure. (4 Marks)

d) Determine bending moments at B, C and D. (6 Marks)

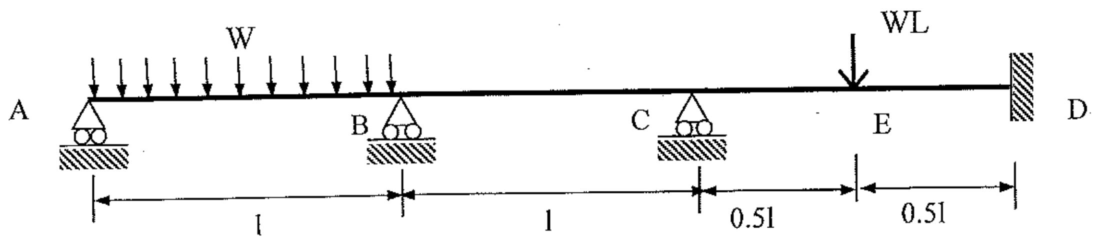


Figure Q4

QUESTION 5

(i) Briefly explain the procedure of the displacement method of structural analysis. (4 Marks)

(ii) Consider the frame structure shown in Figure Q5. Flexural rigidities of members are EI. Find the free nodal displacements at B and C using the displacement method. You can neglect the axial deformation. (10 Marks)

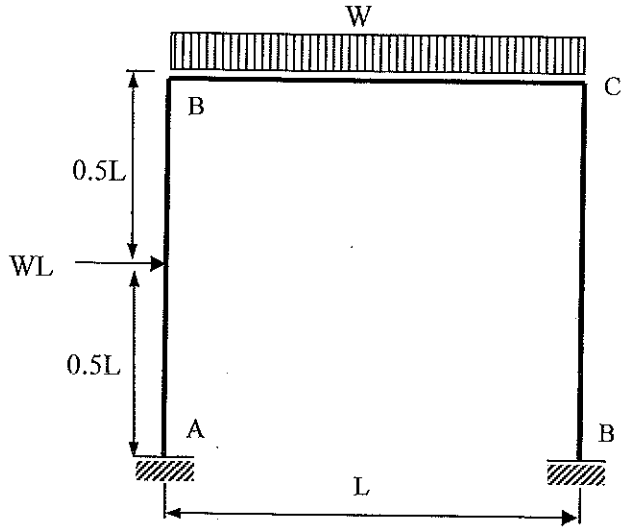


Figure Q5

(iii) Using above results, determine the bending moments at A and B. (6 Marks)

QUESTION 6

(i) Briefly describe following definitions used in the membrane theory of thin shells.

(a) Shell normal

(b) Normal section

(c) Principal radii of curvature (3 Marks)

(ii) A spherical dome with radius '*a*' is supported with a uniformly distributed surface load '*W*' on its plane as shown in Figure Q6. The shell thickness is '*h*'.

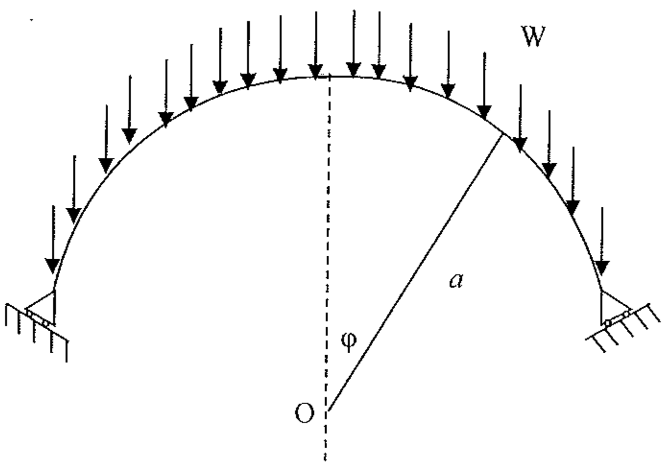


Figure Q6

(a) Derive expressions for meridianal and hoop stresses in the sphere. (9 Marks)

(b) Show that the practical limitation of angle φ is 51.8^0. (5 Marks)

(iii) If the roller supports were changed to fixed supports, discuss the changes in structural response of the shell. (3 Marks)
(No need to analyze and only qualitative answer is sufficient)

QUESTION 7

(i) Describe conceptual difference between elastic design and plastic design of beams and frames.

(3 Marks)

(ii) A two-bay frame structure is loaded as shown in Figure Q7. Dimensions and plastic moment capacity of members are given in the figure.

(a) Determine number of independent failure mechanisms (3 Marks)

(b) Draw failure mechanisms. (3 Marks)

(c) Determine load factors for each failure mechanism. (7 Marks)

(d) Determine the most probable failure mechanism. (4 Marks)

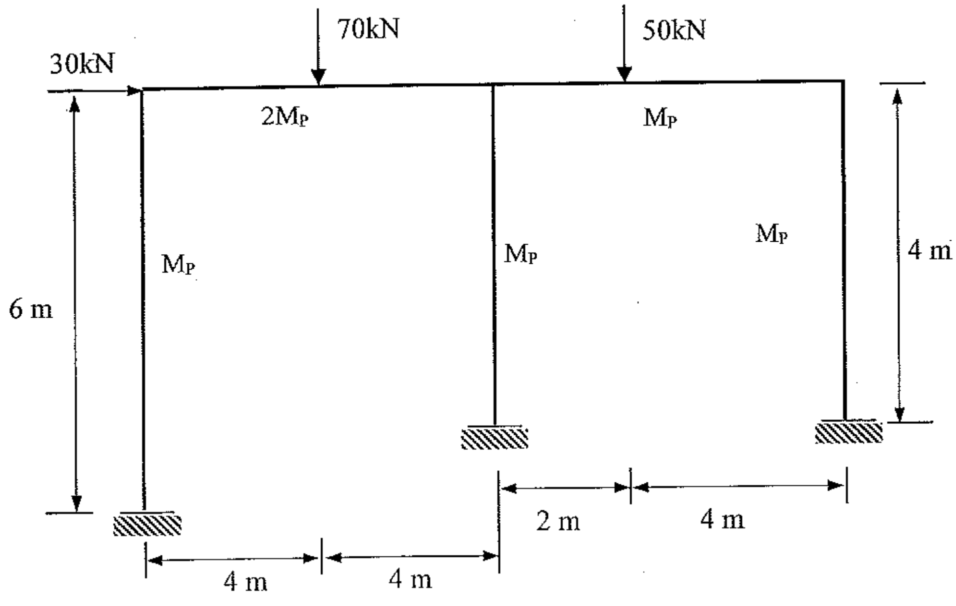


Figure Q7

QUESTION 8

(i) List three assumptions used the analysis of thin plates with small deflections. (3 Marks)

(ii) A solid circular thin plate with a radius "*r*" is subjected to a uniformly distributed load "*q*". The plate has symmetrical boundary condition at the edge.

(a) Obtain a general expression for the plate deflection. (6 Marks)

(b) If the plate is fixed at the edge, determine the expression for deflection. (4 Marks)

(c) Obtain the maximum deflection of the plate. (4 Marks)

(iii) If the plate is thick, describe the changes in deflection obtained above. (3 Marks)

Note: A circular plate of radius "*r*" loaded with uniformly distributed load "*q*", the deflection "*w*" is given with standard notation as

$$\nabla^4 w = \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q}{D}$$

Table 1 Formulas for Beams

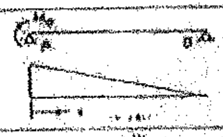
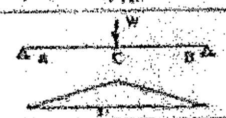
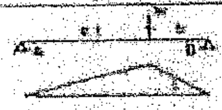
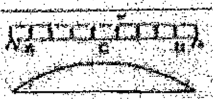
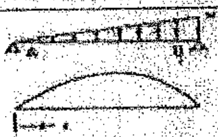
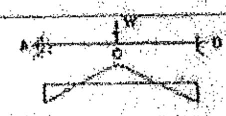
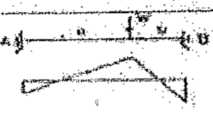
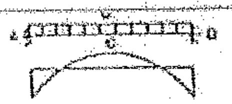
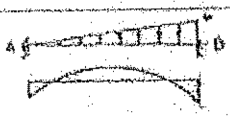
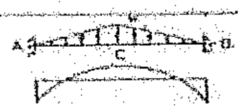
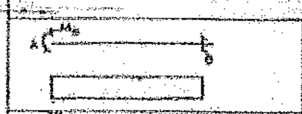
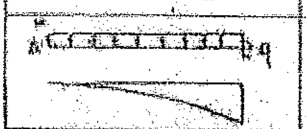
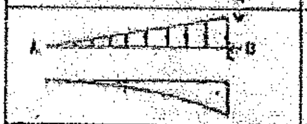
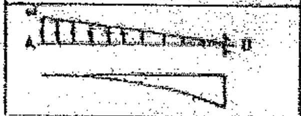
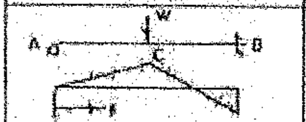
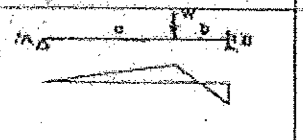
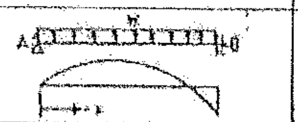
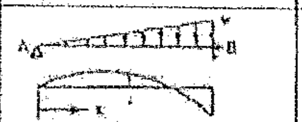
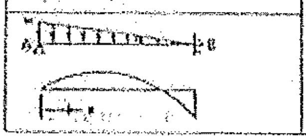
Structure	Shear	Moment	Slope	Deflection
Simply supported Beam				
	$S_A = -\frac{M_e}{L}$	M_e	$\theta_A = \frac{M_e L}{3EI}$, $\theta_B = -\frac{M_e L}{6EI}$	$Y_{max} = 0.062\frac{M_e L^2}{EI}$ at $x = 0.422L$
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{4}$	$\theta_A = -\theta_B = \frac{WL^2}{16EI}$	$Y_c = \frac{WL^3}{48EI}$
	$S_A = \frac{Wb}{L}$, $S_B = \frac{Wa}{L}$	$M_c = \frac{Wab}{L}$	$\theta_A = \frac{Wab}{6EIL}(L+b)$, $\theta_B = -\frac{Wab}{6EIL}(L+a)$	$Y_c = \frac{Wa^2b^2}{3EIL}$
	$S_A = \frac{WL}{2}$	$M_c = \frac{WL^2}{8}$	$\theta_A = -\theta_B = \frac{WL^3}{24EI}$	$Y_c = \frac{5WL^4}{384EI}$
	$S_A = \frac{WL}{6}$, $S_B = \frac{WL}{3}$	$M_{max} = 0.064WL^2$ at $x = 0.577L$	$\theta_A = \frac{7WL^3}{360EI}$, $\theta_B = -\frac{8WL^3}{360EI}$	$Y_{max} = 0.00652\frac{WL^4}{EI}$ at $x = 0.519L$
	$S_A = \frac{WL}{4}$	$M_c = \frac{WL^2}{12}$	$\theta_A = -\theta_B = \frac{5WL^3}{192EI}$	$Y_c = \frac{WL^4}{120EI}$
Fixed Beams				
	$S_A = \frac{W}{2}$	$M_c = \frac{WL}{8}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^3}{192EI}$
	$S_A = \frac{Wb^2}{L^3}(3a+b)$, $S_B = \frac{Wa^2}{L^3}(3b+a)$	$M_A = -\frac{Wab^2}{L^2}$, $M_B = -\frac{Wba^2}{L^2}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{Wa^3b^3}{3EIL^3}$
	$S_A = \frac{WL}{2}$	$M_A = M_B = -\frac{WL^2}{12}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{WL^4}{384EI}$
	$S_A = \frac{3WL}{20}$, $S_B = -\frac{7WL}{20}$	$M_A = -\frac{WL^2}{30}$, $M_B = -\frac{WL^2}{20}$	$\theta_A = \theta_B = 0$	$Y_{max} = 0.00131\frac{WL^4}{EI}$ at $x = 0.525L$
	$S_A = \frac{WL}{4}$	$M_A = M_B = -\frac{5WL^2}{96}$	$\theta_A = \theta_B = 0$	$Y_c = \frac{0.7WL^4}{384EI}$

Table 1 Formulas for Beams

Structure	Shear	Moment	Slope	Deflection
		Cantilever Beam		
	0	M_o	$\theta_A = \frac{M_o L}{EI}$	$Y_A = \frac{M_o L^2}{2EI}$
	W	$M_B = -WL$	$\theta_A = -\frac{WL^2}{2EI}$	$Y_A = \frac{WL^3}{3EI}$
	$S_B = -WL$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{6EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{6}$	$\theta_A = -\frac{WL^3}{24EI}$	$Y_A = \frac{WL^4}{8EI}$
	$S_B = -\frac{WL}{2}$	$M_B = -\frac{WL^2}{2}$	$\theta_A = -\frac{WL^3}{8EI}$	$Y_A = \frac{11WL^4}{120EI}$
		Propped Cantilever		
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{M_o}{2}$	$\theta_A = -\frac{M_o L}{4EI}$	$Y_{max} = \frac{M_o L^2}{27EI}$ at $x = \frac{L}{3}$
	$S_A = -\frac{3M_o}{2L}$	$M_B = -\frac{3WL}{16}$ $M_c = \frac{5WL}{32}$	$\theta_A = \frac{WL^2}{32EI}$	$Y_{max} = 0.00962\frac{WL^3}{EI}$ at $x = 0.447L$
	$S_A = \frac{Wb^2}{2L^3}(a+2L)$ $S_B = -\frac{Wa}{2L^3}(3L^2 - a^2)$	$M_B = -\frac{Wab}{L^2}(a+\frac{b}{2})$	$\theta_A = \frac{Wab^2}{4EIL}$	$Y_0 = \frac{Wa^2 b^3}{12EIL^3}(3L+a)$
	$S_A = +\frac{3WL}{8}$	$M_B = -\frac{WL^2}{8}$	$\theta_A = \frac{WL^3}{48EI}$	$Y_{max} = 0.0054\frac{WL^4}{EI}$ at $x = 0.422L$
	$S_A = +\frac{WL}{10}$	$M_{max} = 0.03WL^2$ at $x = 0.447L$ $M_B = -\frac{WL^2}{15}$	$\theta_A = \frac{WL^3}{120EI}$	$Y_{max} = 0.00239\frac{WL^4}{EI}$ at $x = 0.447L$
	$S_A = +\frac{11WL}{40}$	$M_{max} = 0.0423WL^2$ at $x = 0.329L$ $M_B = -\frac{7WL^2}{120}$	$\theta_A = \frac{WL^3}{80EI}$	$Y_{max} = 0.00305\frac{WL^4}{EI}$ at $x = 0.402L$