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The Open University of Sri Lanka
Faculty of Engineering Technology

Study Programme	:	Bachelor of Technology Honours in Engineering
Name of the Examination	:	Final Examination
Course Code and Title	**:**	**DMX5533 Dynamics of Mechanical Systems**
Academic Year	:	2017/18
Date	:	22nd January 2019
Time	:	0930 hours -1230 hours
Duration	:	3 hours

General instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of Eight (08) questions in Six (06) pages.
3. Answer any 05 questions only. All questions carry equal marks.

Question 01 – ***(20 Marks)***

Figure Q01 shows the seismic unit of vibration-measuring device, which consists of a mass 'm' supported from a frame by a spring of stiffness 'k' in parallel with a damper of viscous damping coefficient 'c'.
The frame of the unit is attached to the structure whose vibration is to be determined. The quantity measured being '**z**', where the relative motion between the seismic mass and the frame. The motion of both the structure and the seismic mass is translation in the vertical direction only.

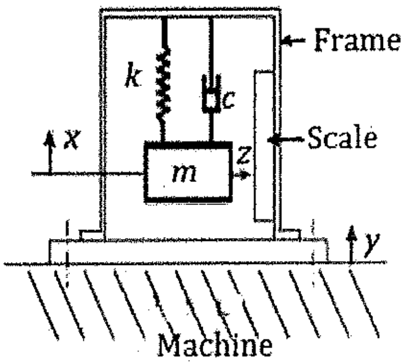

Figure Q01

i. Derive the equation of motion of the seismic mass, assuming that the structure has simple harmonic motion of circular frequency 'ν', and deduce the steady state amplitude of '**z**'.

ii. Given that the undamped natural circular frequency ω of the unit is much greater than ν. Show, why the unit may be used to measure the acceleration of the structure.

iii. Briefly explain why damping is desirable in industrial applications.

iv. Find the necessary value of the damping ratio, if the sensitivity of the unit (that is, the amplitude of z as a multiple of the amplitude of the acceleration of the frame) has the same value when $\nu = 0.2\omega$, when $\nu < \omega$.

Question 02 - *(20 Marks)*

(a) A body mass 'm' is moving downward as shown in Figure Q02. The motion of the mass is controlled by a spring of stiffness 'k' and a dash pot of damping constant 'c' attached to the frame. $F(t)$ is the external force that act on downward direction.

i. Write the equation of motion for the system shown in Figure Q02:
ii. Draw the appropriate free body diagram for zero spring force position.
iii. Draw the appropriate free body diagram for static equilibrium position.

(b) For the system shown in Figure Q02, spring stiffness k =10,000 N/m, **m**=0.633 kg, x_o=0.1m, v_o =10 m/s and the damping ratio β =0.05. (x_o and v_o are initial values)

i. Find an expression for $x(t)$ in terms of the initial conditions. Hence express $x(t)$ in the form of $x(t) = A\cos(\omega t - \emptyset)$.
ii. Sketch $x(t)$ versus time.
iii. Compute the ratio of the damped to the undamped natural frequency for damping ratios of (I) 5%, (II) 10% and (III) 20% of the critical damping.

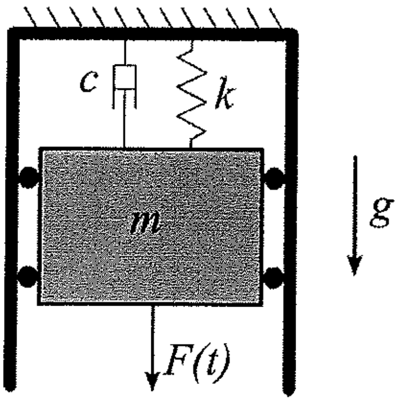

Figure Q02

Question 03 - *(20 Marks)*

Figure Q 03 shows a system of three rotors, the moments of inertia of which are given as I_1 = 100 kg.m^2, I_2 = 150 kg.m^2 & I_3 = 1000 kg.m^2. The shaft between I_1 and I_2 is 1m long and 100 mm diameter, and that between I_2 and I_3 is 0.6 m long and 50 mm diameter. The modulus of rigidity of the material of the shaft is 82 kN/mm^2.

i. Determine the natural frequencies of torsional vibrations and the positions of the nodes in each case.
ii. Draw the mode shapes for two cases

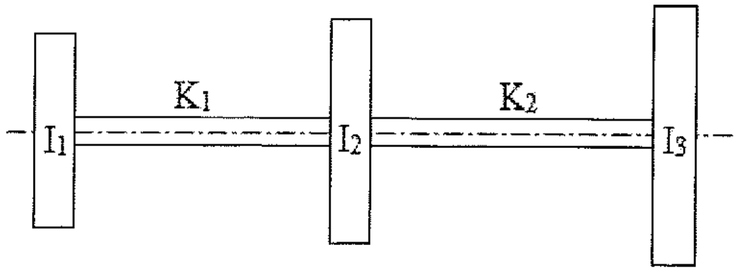

Figure. Q03

Question 04 - *(20 Marks)*

(a) A light horizontal shaft supported in bearings at its ends and carries a rotor of mass 'M' at its span. The center of mass of the rotor is at a distance '*e*' from the shaft axis. Show that the deflection y of the shaft at its mid span when running at an angular speed ω is given by;

$$y = \frac{e}{\left(\frac{\omega_c}{\omega}\right)^2 - 1}$$

where ω_c is the critical speed of the shaft.

(b) A uniform shaft of diameter 25 mm and mass 4.0 kg is supported horizontally in frictionless short bearings. The span of the shaft is 2.0 m. A disc of mass 10 kg is attached to the mid span of the shaft and the disc has an eccentricity of 5.5 mm. Assuming that one half of the mass of the shaft is concentrated at mid span, determine;

i. critical speed of the shaft.
ii. maximum amplitude of vibration.
iii. maximum dynamic force transmitted to each bearing.

Assume;

- The speed of the shaft as 350 rev/min
- The modules of elasticity of the shaft material is 200 GN/m^2

Question 05 - *(20 Marks)*

Compute the transfer function of the block diagram shown in Figure Q05 in following two cases.

(a) Reducing a system to single transfer function and
(b) By plotting the relevant signal flow graph applying Mason's gain formula.

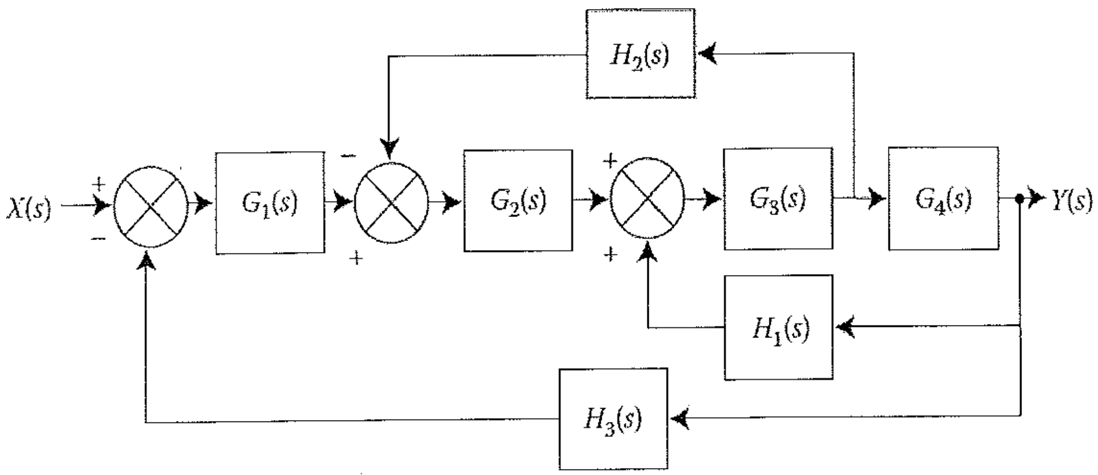

Figure. Q05

Question 06 - ***(20 Marks)***

The Figure Q06 shows a hydraulic servo mechanism. Assuming that;

- all friction (except that intended to be present in the dashpot which has a viscous damping coefficient 'c') are negligible,
- the oil is incompressible,
- the inertia forces are negligible, and
- the volume flow per unit time through either port in the valve is 'b' times the displacement of the valve spool from its neutral position.

(a) Derive the transfer function relating the output displacement '**z**' to the input displacement 'x' (this being measured from the position corresponding to zero oil flow).

(b) Find the value of '**z**' as a function of time following a sudden change in 'x' of magnitude '**X**'.

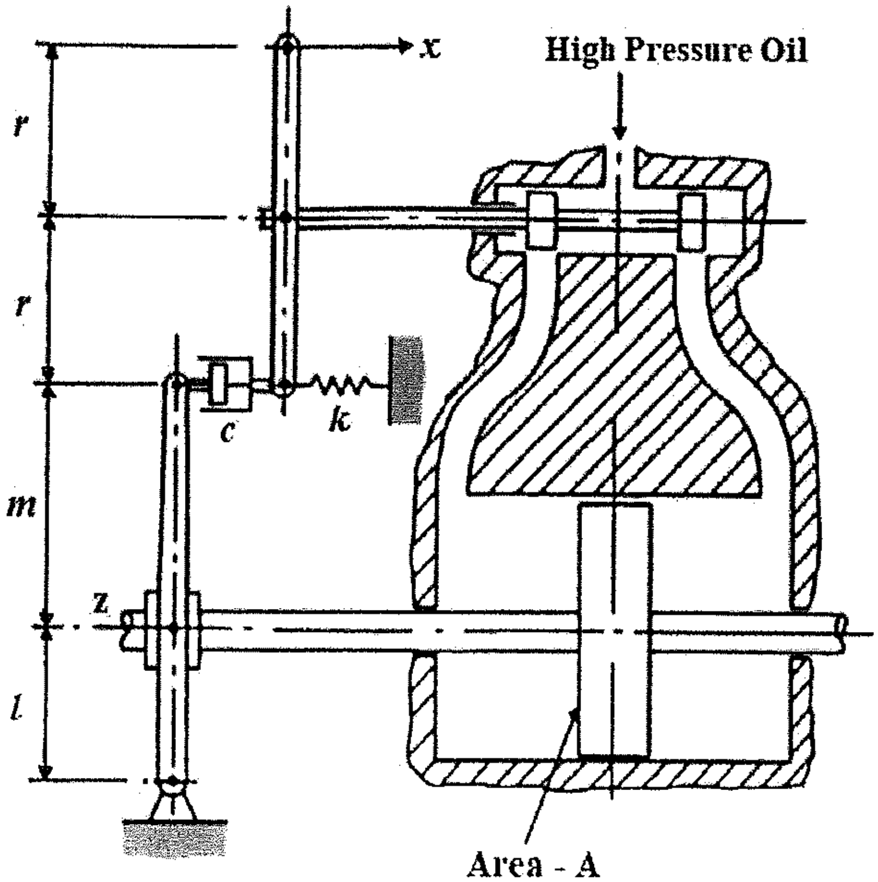

Figure Q 06

Note: *If you have any doubt as to the interpretation of the assumption, make your own decision and clearly state it in the answer script.*

Question 07 - *(20 Marks)*

(a) The system subjected to unit step input on a servomechanism that measurements show the system response to be

$$c(t) = 1 + 0.2\, e^{-60\,t} - 1.5\, e^{-15\,t}.$$

i. Determine the closed loop transfer function
ii. Determine the undamped natural frequency and the damping ratio of the system.

(b) The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{20}{s(s+5)}$$

This system based on a unit step input.
A PD controller having a transfer function Gc(s) = 1 + 0.8s is introduced to the system.
Determine the effect of adding this controller to the system on;

i. peak overshoot,
ii. natural frequency
iii. settling time and
iv. steady state error

Question 08 - *(20 Marks)*

(a) The transfer function of a Proportional and Integral (P + I) controller given below.

$$C(s) = K\left(1 + \frac{1}{T_i s}\right)$$

The unity feedback system to control a process modelled by the transfer function can be shown as

$$G(s) = \frac{1}{1+5s}.$$

Sketch the root locus as controller gain K is increased from a small value for,

i. $T_i = 12$ s
ii. $T_i = 4$ s.

(b) Sketch the root locus of the system for positive values of K with having an open loop transfer function,

$$GH(s) = \frac{K(S+10)}{S(S^2+5S+15)}$$

Determine the range of K for which the system is stable.

LAPLACE TRANSFORMS

TIME FUNCTION f(t)	LAPLACE TRANSFORM F(s)
Unit Impulse $\delta(t)$	1
Unit step	$\frac{1}{s}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{df(t)}{dt}$	sF(s)-f(0)
$\frac{d^n f(t)}{dt^n}$	$s^n F(s) - s^{n-1} f(0) - s^{n-2}\frac{df(0)}{dt} \dots - \frac{d^{n-1} f(0)}{dt^{n-1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$