22 00092

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme	: Bachelor of Technology Honours in Engineering
Name of the Examination	: Final Examination
Course Code and Title	**: EEX5533 / ECX5233**
	Communication Theory & Systems
Academic Year	: 2017/18
Date	: 12th February 2019
Time	: 9.30-12.30hrs

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **Five (5)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. This is a Closed Book Test (CBT).
6. Answers should be in clear hand writing.
7. Do not use Red colour pens.

Q1.

a) A signal $g(t)$ is given by $g(t) = (2cos\ \omega t)(1 + cos\ \omega t)$.
   i. Show that $g(t)$ is a periodic signal. **(3 Marks)**
   ii. Determine the period of the signal $g(t)$. **(3 Marks)**
   iii. Express $g(t)$ as a Fourier series and find the Fourier coefficients. **(3 Marks)**

b) Let $h(t) = g(t) + jsin\ 2\omega t$,
   i. Express $h(t)$ as a Fourier series using the results of a) ii. **(2 Marks)**
   ii. Sketch the amplitude and phase spectrums of $h(t)$. **(2x3 Marks)**

c) Let $k(t) = h(t - t_0)$. Show that the amplitude spectrum of $k(t)$ is identical to that of $h(t)$. **(3 Marks)**

Q2.

a) Fourier transform of a time domain signal $x(t)$ is given by $X(f)$. Using the first principles find the Fourier transforms of the following functions in terms of $X(f)$. **(3x2 Marks)**

i. $Ax(t)$where A is a constant
ii. $x(t - t_0)$ for a t_0 time shift
iii. $\frac{dx(t)}{dt}$

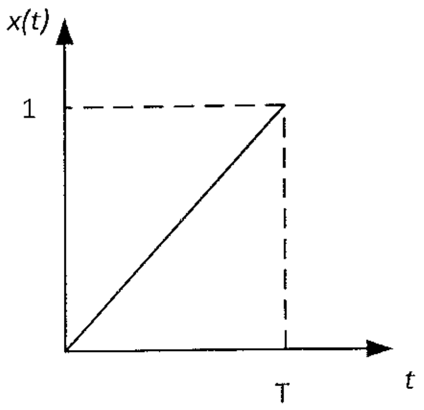

Figure Q2 (a)

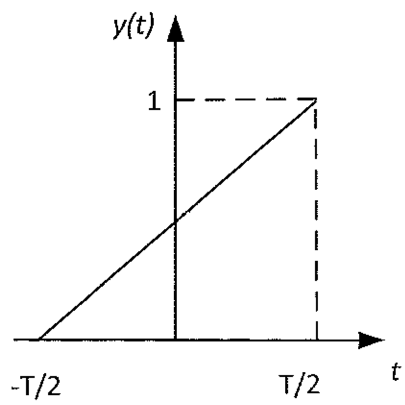

Figure Q2 (b)

b) Let $x(t)$ be the waveform given in Figure Q2 (a). Using first principles find the Fourier transform of $x(t)$ given by $X(f)$. **(6 Marks)**

c) Using the results of a) and b) find the Fourier transform of the waveform in Figure Q2 (b). **(3 Marks)**

d) Hence find the Fourier transform of the square pulse train waveform in Figure Q2 (c). **(5 Marks)**

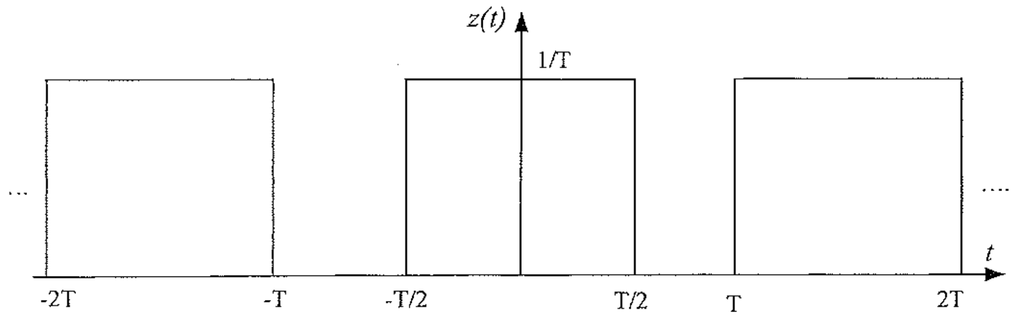

Figure Q2 (c)

Q3.

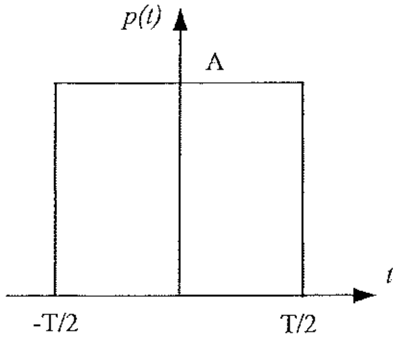

Figure Q3 (a)

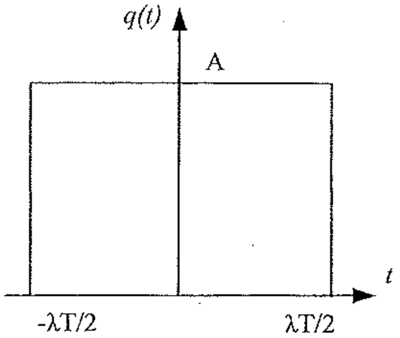

Figure Q3 (b)

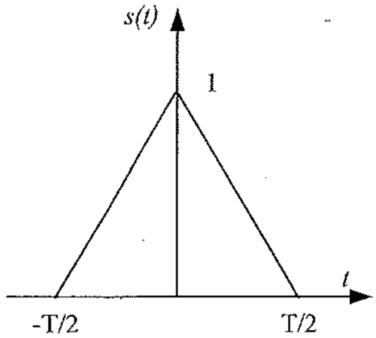

Figure Q3 (c)

a) Consider two time domain functions $p(t)$ and $q(t)$ given in Figure Q3 (a) and (b). Let $\lambda \geq 1$. Find the convolution function $r(t) = p(t) * q(t)$. **(6 Marks)**

b)

i. Find the Fourier transform of $p(t)$ given by $P(f)$. **(4 Marks)**

ii. Hence find the Fourier transform of $q(t)$ given by $Q(f)$. **(2 Marks)**

iii. Find and sketch $R(f)$ for $\lambda = 2$. **(4 Marks)**

c) Using the results of b) find the Fourier transform of $s(t)$ given by $S(f)$. **(4 Marks)**

Q4.

a) Using the first principles derive the Fourier transform of a Dirac Delta impulse signal $\delta(t)$. **(4 Marks)**

b) Hence find the Fourier transform of an impulse train $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. **(4 Marks)**

c) A signal $f(t)$ with a bandwidth W is sampled using the impulse train $s(t)$ where the sampled signal is given by $f_s(t) = f(t).s(t)$.

i. Sketch the signal $f_s(t)$. **(2 Marks)**

ii. Find $F_s(\omega)$ the Fourier transform of $f_s(t)$ in terms of the Fourier transform of $f(t)$ given by $F(\omega)$. **(4 Marks)**

iii. Show that $F(\omega)$ cannot be recovered from $F_s(\omega)$ without distortion if $2W > \frac{1}{T_s}$. **(2 Marks)**

iv. Find the amplitude response of a suitable filter which can recover $F(\omega)$ from $F_s(\omega)$ when $2W \leq \frac{1}{T_s}$. **(4 Marks)**

Q5.

a) A carrier $c(t)$ is full amplitude modulated using a signal $m(t)$ and a modulation index of 1.

i. Write an expression for the modulated signal $s(t)$ in terms of $m(t)$ and $c(t)$. **(2 Marks)**

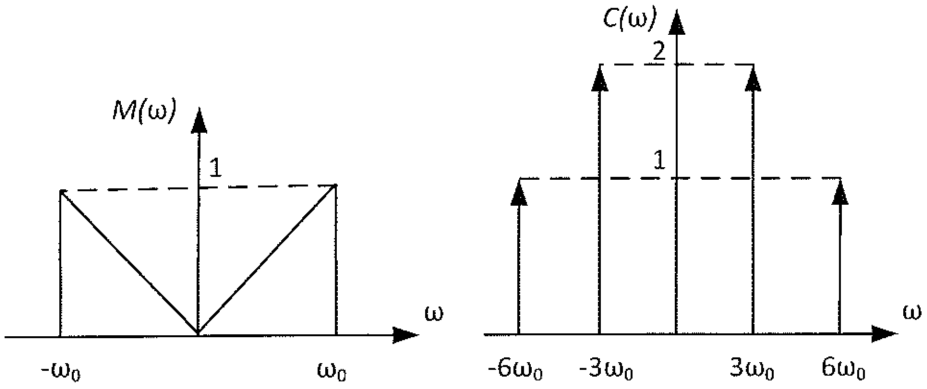

Figure Q5

ii. Fourier transforms of $m(t)$ and $c(t)$ are $M(\omega)$ and $C(\omega)$ respectively as given in Figure Q5. Find the Fourier transform of $s(t)$, $S(\omega)$ and sketch $S(\omega)$. **(4 Marks)**

iii. If $s(t)$ is transmitted over an ideal channel after suppressing the carrier, write an expression for the received signal $y(t)$ in terms of $m(t)$. **(2 Marks)**

iv. Explain how would you recover $m(t)$ from $y(t)$. **(2 Marks)**

v. Explain how would you recover $m(t)$ if the channel is having ideal low-pass characteristics with a cut-off frequency $2\pi\omega_0$. **(4 Marks)**

b) A phase modulated signal is given by $x_c(t) = \cos(\omega_c t + \beta \sin \omega_m t)$ where ω_c is the carrier's angular frequency. Using a proper mathematical calculation find the bandwidth of this signal for $\beta \ll 1$. **(6 Marks)**

Q6.

a) Let $x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$. Express $x(t)$ as a Fourier series and find the coefficients. **(4 Marks)**

b) A communication system has a low pass characteristics given by the frequency response ,

$$H(\omega) = \begin{cases} e^{-\frac{\omega^2}{\omega_B}} & |\omega| \leq 2\omega_B \\ 0 & \text{othersise.} \end{cases}$$

i. If $x(t)$ in a) is input to this communication system sketch the frequency response of the output signal. $[\omega_B = 4.\frac{2\pi}{T_0}]$ **(6 Marks)**

ii. Calculate the power of the output signal. **(4 Marks)**

c) If $x(t)$ in a) is transmitted through a system having an impulse response $g(t)$ given by,

$$g(t) = \begin{cases} 1 & |t| \leq \frac{T_0}{4} \\ 0 & \text{othersise.} \end{cases}$$

Calculate and sketch the output signal. **(6 Marks)**

Q7.

a) Using appropriate diagrams explain the term *ensemble average of a random signal.* **(2 Marks)**

b) State the conditions to be satisfied for a random signal to be

ii. Wide sense stationary (WSS)

iii. Ergodic **(2x3 Marks)**

c) A certain random signal is given by $x(t) = at + b$ where a is a random variable uniformly distributed in the range $[-1, 1]$ and b is a constant.

i. Determine the ensemble average $\overline{x(t)}$. **(4 Marks)**

ii. Determine the autocorrelation $R_X(t_1, t_2)$. **(6 Marks)**

iii. Is this signal wide sense stationary? Reason your answer. **(2 Marks)**

Q8.

a) A certain document consists of only five symbols $\alpha, \beta, \gamma, \delta$ and ε. The probabilities of the occurrence of the symbols are given by $\Pr(x = \alpha) = 0.3$, $\Pr(x = \beta) = 0.15$, $\Pr(x = \gamma) = 0.15$, $\Pr(x = \delta) = 0.35$ and $\Pr(x = \varepsilon) = 0.05$.

i. Find the entropy of the symbols. **(3 Marks)**

ii. Using Huffman technique design a coding scheme / codeword set to have a minimum average codeword length to represent a symbol. **(4 Marks)**

iii. Calculate the minimum average codeword length. **(2 Marks)**

b) Binary erasure channel (BEC) is a common communication channel model which has two inputs and three outputs as shown in Figure Q8. Status 'e' is known as 'erased' which represents a corrupted bit. Let $p(x = 0) = \pi$.

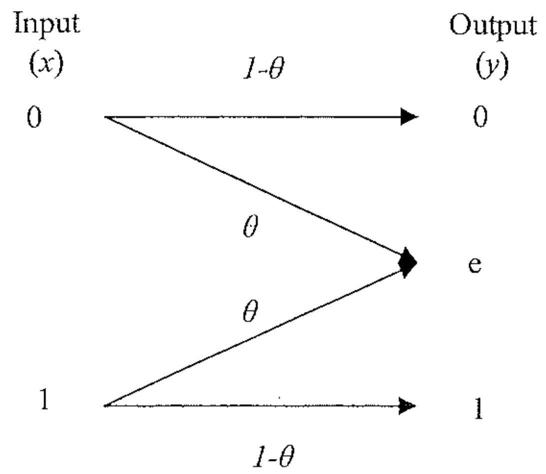

Figure Q8

i. Write down an expression for the capacity of the channel in terms of the mutual information between the input and output of the channel. **(1 Marks)**

ii. Write down an expression for the entropy of the output of the channel given the input. **(3 Marks)**

iii. Find the maximum possible entropy of the input x for varying π. **(3 Marks)**

iv. Hence show that the capacity of the BEC is $1 - \theta$. **(4 Marks)**