00070

(7)

The Open University of Sri Lanka
Faculty of Engineering Technology
Department of Electrical & Computer Engineering

Study Programme : Diploma in Information Systems and Technology
Name of the Examination : Final Examination
Course Code and Title : EEZ3562/ECZ3262 Mathematics
Academic Year : 2017/18
Date : 5th February 2019
Time : 0930hrs - 1230hrs
Duration : **3 hours**

General Instructions

1. Read all the instructions carefully before answering the questions.
2. This question paper consists of **Eight (8)** questions in **Five (5)** pages.
3. Answer any **Three (3)** questions in **Section A** and any **Two (2)** questions in **Section B**.
4. Answer for each question should commence from a new page.
5. Show **intermediate steps** clearly.
6. This is a Closed Book Test (CBT).
7. Answers should be in clear hand writing.
8. Do not use Red colour pen.

SECTION A

Q1

(a) If P, Q and R are Boolean variables, then construct a truth table for $(P \rightarrow Q) \wedge (Q \rightarrow R)$.

[4 Marks]

(b) Using the results of boolean algebra, minimize the following expressions.

(i) $(a + b)(a + b') = a$. **[3 Marks]**

(ii) $(a(b + z(x + a')))' = a' + b'(z' + x')$ **[3 Marks]**

(c) If R, S, T, U and F are Boolean variables, then

(i) Simplify the following truth table by using Karnaugh map **[5 Marks]**

(ii) Write down the simplified expression. **[5 Marks]**

R	S	T	U	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

Q2

(a) Prove that the matrix $\begin{bmatrix} -i & 2+i \\ -2+i & 0 \end{bmatrix}$ is a skew-Hermitian matrix. **[4 Marks]**

(b)

(i) If $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 1 & 0 \end{bmatrix}$ show that $AB \neq BA$. **[3 Marks]**

(ii) If $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 3 & 2 \\ -1 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}$ find $2A - 3B$. **[3 Marks]**

(c) Show that the matrix $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ is the inverse matrix of the matrix

$\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$. **[10 Marks]**

Q3

(a) By using the integration by parts, find the indefinite integral **[6 Marks]**

$\int x^2 e^{3x} dx$.

(b) Evaluate following the definite integrals. **[8 Marks]**

(i) $\int_0^{\frac{\pi}{3}} \frac{\cos x + \cos x \tan^2 x}{2\sec^2 x} dx$ (ii) $\int_0^2 x(x^2 - 1)^7 dx$

(c) Find the partial fractions of $\frac{3}{(x-1)(x+1)^2}$.

[6 Marks]

Hence, find the indefinite integral $\int \frac{3}{(x-1)(x+1)^2} dx$.

Q4

(a) Evaluate the limit $\lim_{x \to 0} \frac{\cos x - 1}{x}$ **[4 Marks]**

(b) Find the first derivative of each of the following functions.

(i) $f(x) = 4x^4 - 2x + 3\tan x + 7$ **[4 Marks]**

(ii) $f(x) = \frac{4x^2 - 2x + 7}{x^3}$ **[4 Marks]**

(iii) $f(x) = [\ln \sqrt{x+1}]$ **[4 Marks]**

(iv) $f(x) = \sin 5x^3 + 2x$ **[4 Marks]**

Q5

(a) A pulley belt of length 300 cm takes 2 s to make a complete revolution. If the radius of the pulley is 150 mm, then find the angular velocity of a point on the rim of the pulley.

[4 Marks]

(b) Find the value of

(i) Sin x + Sin 2x + Sin 4x, when $x = 60^0$ **[4 Marks]**

(ii) Cos 4x – Cos 3x + Cos x, when $x = 120^0$ **[4 Marks]**

(c) Prove that

$$\frac{Tan\theta}{1+Sec\theta}-\frac{Tan\theta}{1-Sec\theta}=2Co\sec\theta.$$ **[8 Marks]**

SECTION B

Q6

(a) Let $A=\begin{bmatrix}1 & 4 & 5\\ 0 & 3 & 2\\ -1 & 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 0\\ 0 & 1 & 2\\ 3 & 0 & 2\end{bmatrix}$. Find $A^{T}B^{T}$ **[4 Marks]**

(b) Convert the following angles to given units.

(i) 102.4761^0 to Minutes, Seconds form. **[3 Marks]**

(ii) $57^0 27' 6''$ to decimal form. **[3 Marks]**

(c) Prove the following identity. **[10 Marks]**

$$\frac{(\tan x+\sec x-1)}{(\tan x-\sec x+1)}=\frac{(1+\sin x)}{\cos x}$$

Q7

(a)

(i) By using the differentiation of the first principles, find the gradient of $f(x) = 5x^2 - 3x + 7$ at the point $x = -2$. **[4 Marks]**

(ii) Evaluate the limit $\lim_{x \to 0} \frac{\sin(\frac{\pi}{4} + x) - \sin(\frac{\pi}{4})}{x}$ **[4 Marks]**

(b) Find the first derivative of each of the following functions. **[6 Marks]**

(i) $f(x) = \frac{x \cos ec(x)}{3 - \cos ec(x)}$

(ii) $f(x) = (2e^{x^2} + x^2)^3$

(c) Find the equation of the line perpendicular to the graph of $y = \frac{\tan x}{1 + \tan x}$ at the point $x = \frac{\pi}{4}$. **[6 Marks]**

Q8

(a)

(i) Using Newton's backward interpolation formula and the following table, calculate an approximation value for f (7.5) correct to 9 decimal places.

[8 Marks]

x	1	2	3	4	5	6	7	8
y=f(x)	1	8	27	64	125	216	343	512

(ii) Assume that the equation $x^3 - 2x - 5 = 0$ has a root near 2. Using Newton-Raphson method, calculate an approximation value for the above root correct to 9 decimal places. **[8 Marks]**

(b) A girl thinks that she can read 240 pages of a book every day. However, in one week (7 days) she has read only 200 pages. Calculate the percentage error.

[4 Marks]

-END-