The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination 2016/2017
Applied Mathematics – Level 05

APU 3145/ APE5145 – Newtonian Mechanics II

Duration :- Two Hours

Date :-11.01.2018	**Time:-01.30 p.m. To 03.30 p.m.**

Answer Four Questions Only.

1. (a) In the usual notation, show that in cylinderical polar coordinates, the velocity and acceleration of a particle are given respectively by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$, .

   (b) A particle of mass m moves on the smooth inside surface of a parabola of revolution $r^2 = 4az,$ whose axis is vertical with vertex downwards. The path of the particle lies exactly between $z = \alpha$ and $z = \beta$. Show that the angular momentum about the z-axis is $m(8az\alpha\beta)^{\frac{1}{2}}$ and that its speed is $\left(2g(\alpha + \beta - z)\right)^{\frac{1}{2}}$.
   Also, show that the reaction between the particle and the surface when $z = \alpha$ is $\dfrac{mg(a+\beta)}{\left(a(\alpha+\beta)\right)^{\frac{1}{2}}}$.

2. (a) State D' Alembert's principle.

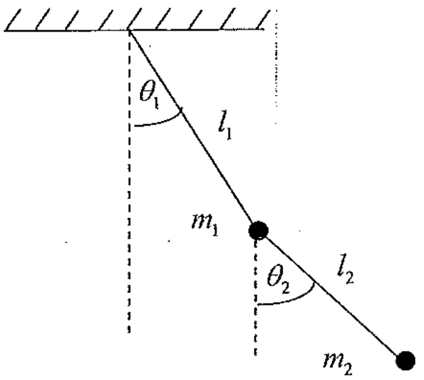
   (b) Two uniform spheres, each of mass M and radius a, are firmly fixed to the ends of two uniform rods, each of mass m and length l, and the other ends of the rods are freely hinged to a point O. The whole system revolves about a vertical line through O with the angular velocity ω. Show that when the motion is steady, the rods are inclined to the vertical at an angle θ given by the equation $\cos\theta = \dfrac{g\left\{M(l+a) + (1/2)ml\right\}}{\omega^2\left\{M(l+a)^2 + (1/3)ml^2\right\}}$.

3. (a) Obtain, in the usual notation, the equation $\dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \dfrac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) A projectile located at a point of latitude λ is projected with speed v_0 in a southward direction at an angle α to the horizontal. Choosing a suitable set of axes, write down the equation of motion for the projectile. Use it to find the position of the projectile after time t. Prove that after time t, the projectile will be deflected towards the east of the original vertical plane of motion by the amount $\frac{1}{3}\omega g\cos\lambda t^3 - \omega v_0 \sin(\alpha+\lambda)t^2$ approximately.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0,\ \ j=1,2\ldots,n.$

(b) The double pendulum consists of two bobs of masses m_1 and m_2 at ends of two weightless rods of lengths l_1 and l_2 and one of them is fixed to a rigid support as shown in figure.



(i) Show that the kinetic energy T of the system is given by

$$T=\frac{1}{2}(m_1+m_2)l_1^2\dot{\theta}_1^2+\frac{1}{2}m_2l_2^2\dot{\theta}_2^2+m_2l_1l_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)$$

(ii) Show that the potential energy V of the system is given by

$$V=-(m_1+m_2)gl_1\cos\theta_1-m_2gl_2\cos\theta_2$$

(iii) Hence obtain the Lagrangian of the system.

(iv) Show that the Lagrange's equations of motion can be written as

$$(m_1+m_2)l_1^2\ddot{\theta}_1+m_2l_1l_2\ddot{\theta}_2\cos(\theta_2-\theta_1)-m_2l_1l_2\dot{\theta}_2^2\sin(\theta_2-\theta_1)+(m_1+m_2)gl_1\sin\theta_1=0 \text{ and}$$

$$m_2l_2^2\ddot{\theta}_2+m_2l_1l_2\ddot{\theta}_1\cos(\theta_2-\theta_1)+m_2l_1l_2\dot{\theta}_1^2\sin(\theta_2-\theta_1)+m_2gl_2\sin\theta_2=0.$$

5. (a) Derive Euler's equations of motion for a rigid body rotating about a fixed point.

(b) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body then show that H and T are conserved.

(c) If a rectangular parallelepiped with its edges $2a, 2a, 2b$ rotates about its center of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and is periodic about the other axis.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i$, $\frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) The Hamilton's of a dynamical system is given by

$$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$$

where a, b are constants. Find q_1, q_2, p_1 *and* p_2 at time t.