**THE OPEN UNIVERSITY OF SRI LANKA**

**B.Sc. /B.Ed. Degree Programme**

**FINAL EXAMINATION 2016/2017**

**APPLIED MATHEMATICS-LEVEL 05**

**APU3146 – OPERATIONAL RESEARCH**

**DURATION: TWO HOURS**

**Date: 16.01.2018** **Time: 09.30 a.m- 11.30 a.m**

**ANSWER FOUR QUESTIONS ONLY.**

**Question 01**

(a) Consider the following payoff matrix for 2×2 two-person zero-sum game which does not have any saddle point.

*Player B*

*Player A*

| | $B_1$ | $B_2$ |
|---|---|---|
| $A_1$ | $a_{11}$ | $a_{12}$ |
| $A_2$ | $a_{21}$ | $a_{22}$ |

(i) Write down the formulas for optimum mixed strategies of Player $A$ and Player $B$ and the value of the game.

(ii) Prove that if a fixed positive number M is added to each element of the above pay-off matrix, then the optimal strategies remain unchanged while the value of the game increases by M.

(b) There are two players in a game, say player $A$ and player $B$. Player $A$ has Rs.2 coin and Rs.5 coin, and player $B$ has Rs.1 coin and Rs.10 coin. Each player selects a coin from the other player without knowing what coin the other player has selected. If the total rupees of the coins selected is odd, player $A$ gets a payoff worth of the two coins that were selected, but if the total is even, player $B$ gets a payoff worth of the two coins.

(i) Construct the payoff matrix with respect to the player $A$.

(ii) Is there a saddle point? Justify your answer.

(iii) Determine the optimal strategies for player $A$ and player $B$.

(iv) Find the value of the game.

[Turn over

**Question 02**

(a) Define the following terms:

(i) Mixed strategy.

(ii) Payoff matrix

(iii) Two-person zero-sum game

(iv) Saddle point

(b) Show that, the inequality $\max \min(a_{ij}) \leq \min \max(a_{ij})$ satisfy for the payoff matrix $a_{ij}$ for a two-person zero-sum game.

(c) Determine the ranges of values of λ and μ that will make the position (2, 2) a saddle point for the game having the payoff matrix given below:

| | | Player *B* | | |
|---|---|---|---|---|
| | | $B_1$ | $B_2$ | $B_3$ |
| Player *A* | $A_1$ | 1 | 3 | 5 |
| | $A_2$ | 8 | 4 | λ |
| | $A_3$ | 2 | μ | 9 |

**Question 03**

(a) What do you mean by transient state and steady state in queueing systems?

(b) A self-service store employs one cashier at its counter. Nine customers arrive on an average every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find

(i) the probability that a customer arriving at the shop will have to wait,

(ii) the average length of the queue that forms.

(iii) the average time a customer spends in the system.

(iv) the probability that there will be three or more customers waiting for the service.

(v) the fraction of the time that there are no customers.

(vi) the average service time need to be decreased to keep the average time in the system less than 3 minutes.

**Question 04**

A group of users in a computer browsing centre has 2 terminals. The average computing job requires 20 minutes of terminal time and each user requires some computation about once every half an hour. Assume that the arrival rate is Poisson and service rate is exponential and the group contains 6 users.

(i) Find the probability that no customer is in the system.

(ii) Find the average number of users waiting to use one of the terminals and in the computing job.

(iii)Find the average waiting time of a user in the queue.

**Question 05**

(a) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and instantaneous supply.

(b) A company makes bicycles. It produces 450 bicycles a month. It buys the tires for bicycles from a supplier at a cost of Rs.2000 per tire. The company's inventory carrying cost is estimated to be 15% of cost and the ordering is Rs.50 per order.

(i) Calculate the EOQ (Economic Order Quantity).

(ii) What is the number of orders per year?

(iii) Compute the average annual ordering cost.

(iv) Compute the average inventory.

(v) What is the average annual carrying cost?

(vi) Compute the total cost.

**Question 06**

(a) Formulate the Economic Order Quantity (EOQ) model in which demand is uniform and replenishment rate is finite.

(b) A Carpet factory manufactures carpets at the rate of 150 yards of carpet per day. Annual demand is 10000 yards of carpet and annual carrying cost is Rs.0.75 per yard. Setup cost is Rs.150 per run. The manufacturing facility operates the same days the store is open, (i.e., 311 days).

[Turn over

Using the formula obtained in part(a) answer to the following questions:

(i) determine the optimum order quantity for one run,

(ii) find the length of production run,

(iii)find the number of runs per year, and

(iv)determine minimum total cost for one run.

## Formulas (in the usual notation)

### (M/M/1):(∞/FIFO) Queuing System

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \left(1-\frac{\lambda}{\mu}\right) \qquad P(\text{queue size} \geq n) = \rho^n$$

$$E(n) = \frac{\lambda}{\mu - \lambda} \qquad E(m) = \frac{\lambda^2}{\mu(\mu-\lambda)} \qquad E(v) = \frac{1}{\mu-\lambda} \qquad E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

### (M/M/1): (N/FIFO) Queueing System

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}}, & \rho \neq 1 \\ \dfrac{1}{N+1}, & \rho = 1 \end{cases} \qquad E(m) = \frac{\rho^2\left[1-N\rho^{N-1}+(N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$E(n) = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})} \qquad E(w) = E(v) - \frac{1}{\mu} \text{ or } E(w) = \frac{\{E(m)\}}{\lambda'}$$

$$E(v) = {[E(n)]}/{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

### (M/M/C):(∞/FIFO) Queuing System

$$P_n = \begin{cases} \dfrac{1}{n!}\rho^n P_0 & ;1 \leq n \leq C \\ \dfrac{1}{C^{n-C}C!}\rho^n P_0 & ;n > C \end{cases} \qquad E(m) = \frac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^C P_o}{(C-1)!(C\mu-\lambda)^2} \qquad E(n) = E(m) + \frac{\lambda}{\mu}$$

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1} \qquad E(w) = \frac{1}{\lambda}E(m) \qquad E(v) = E(w) + \frac{1}{\mu}$$

[Turn over

**(M/M/C): (N/FIFO) Model**

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ; 0 \le n \le C \\ \dfrac{1}{C^{n-1}C!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ; C < n \le N \end{cases}$$

$$P_0 = \begin{cases} \left[\sum_{n=0}^{C-1} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n + \dfrac{1}{C!}\left(\dfrac{\lambda}{\mu}\right)^C \left\{1-\left(\dfrac{\lambda}{C\mu}\right)^{N-C+1}\right\} \dfrac{C\mu}{C\mu-1}\right]^{-1} & ; \dfrac{\lambda}{C\mu} \ne 1 \\ \left[\sum_{n=0}^{C-1} \dfrac{1}{n!}\left(\dfrac{\lambda}{\mu}\right)^n + \dfrac{1}{C!}\left(\dfrac{\lambda}{\mu}\right)^C (N-C+1)\right]^{-1} & ; \dfrac{\lambda}{C\mu} = 1 \end{cases}$$

$$E(m) = \frac{P_o (C\rho)^C \rho}{C!(1-\rho)^2}\left[1-\rho^{N-C+1} - (1-\rho)(N-C+1)\rho^{N-C}\right] \qquad E(w) = E(v) - \frac{1}{\mu}$$

$$E(n) = E(m) + C - P_0 \sum_{n=0}^{C-1} \frac{(C-n)(\rho C)^n}{n!} \qquad E(v) = {}^{[E(n)]}\!/_{\lambda'}, \quad \text{where } \lambda' = \lambda(1-P_N)$$

**(M/M/R):(K/GD) Model**

$$P_n = \begin{cases} \dbinom{K}{n}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ; \ 0 \le n < R \\ \dbinom{K}{n}\dfrac{n!}{R^{n-R}R!}\left(\dfrac{\lambda}{\mu}\right)^n P_0 & ; \ R \le n \le K \end{cases}$$

$$P_0 = \left[\sum_{n=0}^{R-1}\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \sum_{n=R}^{K}\binom{K}{n}\frac{n!}{R^{n-R}R!}\left(\frac{\lambda}{\mu}\right)^n\right]^{-1}$$

$$E(n) = P_0\left[\sum_{n=0}^{R-1} n\binom{K}{n}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{R!}\sum_{n=R}^{K} n.\binom{K}{n}\frac{n!}{R^{n-R}}\left(\frac{\lambda}{\mu}\right)^n\right] \qquad E(v) = \frac{E(n)}{\lambda[K-E(n)]}$$

$$E(m) = \sum_{n=K}^{K}(n-R)P_n \qquad E(w) = \frac{E(m)}{\lambda[K-E(n)]}$$

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