The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination- 2016/2017
Applied Mathematics-Level 05
APU 3150/APE 5150 Fluid Mechanics

Duration:-Two hours

Date:- 04.01.2018	**Time: 9.30 a.m. -11.30 a.m.**

Answer **FOUR** questions only. Standard notation are used throughout this paper.

1. (a) Briefly distinguish between the two types of fluid flow mentioned below.
   i. Steady/Unsteady flows
   ii. Uniform/Nonuniform flows
   iii. Compressible/Incompressible flows
   iv. Rotational/Irrotational flows.

   (b) The fluid flow field with velocity vector $\mathbf{q} = x^2y\mathbf{i} + y^2z\mathbf{j} - (2xyz + z^2y)\mathbf{k}$, in the usual notation. Verify that **steady, incompressible fluid** motion is possible with the velocity $\mathbf{q}$.

   (c) Show that the velocity vector $\mathbf{q} = e^x[(sinz - cosy)\,\mathbf{i} + siny\,\mathbf{j} + cosz\,\mathbf{k}]$ represents possible **irrotational** motion of an incompressible fluid.

2. (a) Show that $\underline{q} = (-\omega y, \omega x, 0)$, where ω is a constant, represents the velocity of an incompressible fluid in a **rotational motion**, and that streamlines are the circles lying on the cylinders $x^2 + y^2 = a^2$ and the planes $z = c$, where a and c parameters.

   Find the **vorticity vector** in this motion, and show that the vortex lines are parallel to the z- axis.

(b) Show that **equation of continuity** can be reduced to the form $\nabla^2\phi = 0$ for an incompressible fluid in an irrotational motion, where ϕ denotes the velocity potential.

Verify that $\phi = \frac{Cx}{r^3}$, where C is a constant and $r^2 = x^2 + y^2 + z^2$, represents possible motion satisfying the above form of the continuity equation. What would be the fluid velocity, in this motion?

3. A fluid of variable density ρ, is in equilibrium under the external force $\underline{F}$ per unit mass. By considering equilibrium of an arbitrary portion of fluid of volume V bounded by a surface S every element δS of which is acted upon by a **pressure force** $-\underline{n}(p\delta S)$, where $\underline{n}$ is a unit vector in the direction outward to the element δS, obtain the equation $\rho\underline{F} = \text{grad}p$.

[Gauss' divergence theorem for a vector field may be assumed here.]

(a) If $\underline{F} = -g\underline{k}$, where g is the constant of gravitation, and the unit vector $\underline{k}$ points vertically upwards, deduce that $dp = -\rho g dz$.

(b) Furthermore, if $\rho = \rho_0 exp(-z)$ where ρ_0 is the constant density on the free surface, $z = 0$, show that $p = p_0 - \rho_0 g(1 - e^{-z})$, where p_0 is the constant pressure acting on the free surface.

4. A right circular cylinder $r = a$, where $r^2 = x^2 + y^2$, stands with its axis vertical and its base attached to a infinite rigid horizontal plane $z = 0$. It is surrounded by an ocean of incompressible non-viscous liquid of infinite extent, bounded below by the plane $z = 0$ and above by its free surface open to the atmosphere at pressure p_0. The cylinder extends above the free surface of the ocean, whose height at a large distance from the cylinder is h.

Given that the velocity components of the liquid at the point (x, y, z), are $(\frac{\omega a^2 y}{r^2}, -\frac{\omega a^2 x}{r^2}, 0)$, where ω is a constant, show that the motion is irrotational and find the following quantities:

(a) Velocity potential of the motion.

(b) Liquid pressure at a point on the surface of the cylinder at a height z.

(c) Liquid pressure on the plane base $z=0$, at distance $r(>a)$ from the axis.

(d) Height of the free surface above the plane base $z=0$, as it touches the cylinder.

5. Write down, without derivation, Bernoullis equation for unsteady irrotational motion of an incompressible non-viscous liquid.

A spherical bubble of gas is inside an infinite liquid of constant density ρ. Initially (at time $t=0$), its radius is a, pressure of the gas is p_0 and the sphere begins to expand radially, its radius $R(t)$ and pressure p satisfying the relationship $p=p_0(\frac{a}{R})^4$. Assuming the form $\underline{q}=(R^2\dot{R})\frac{e_r}{r^2}$, where $\dot{R}=\frac{dR}{dt}$ for resulting **liquid velocity**, in the region $r\geq a$ show that this motion is irrotational with velocity potential $\phi=(R^2\dot{R})\frac{1}{r}$.

Using Bernoullis equation and the substitution $\ddot{R}=\frac{dQ}{dR}$, where $Q=\frac{\dot{R}^2}{2}$, show further that $\dot{R}^2=\frac{2P_0}{\rho}\{(\frac{R}{a})^3-(\frac{R}{a})^4\}$.

6. (a) Given the complex potential function $f(z)=z^2$, $z\in\mathbb{C}$ find the streamlines and equipotential lines, and verify that they are mutually orthogonal.

(b) Consider the complex potential function $F(z)=U(z+\frac{a^2}{z})$ where U and a are positive constants. Find the stream function and the complex velocity in this motion.

Show that

i. One of the stream lines consists of the real axis, $y=0$, and the circle $r=a$, fluid occupying the region outside this circle.

ii. Velocity vector for large r is of magnitude U, and directed parallel to the $x-$axis.

Find the points on the circle $r=a$ where the pressure is maximum.

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