The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2016/2017
Applied Mathematics - Level 03
APU1140/APE3140 – Vector Algebra

Duration: - Two Hours

Date: 24.07.2017 | Time: 01:00 p.m. – 03:00 p.m.

INSTRUCTIONS TO THE CANDIDATES

- This question paper consists of **FOUR (04)** pages and **SIX (06)** questions. If the part of this paper is missing or not printed properly, please inform the supervisor.
- Answer **FOUR (04)** questions **ONLY**.
- Always start to answer each question in a new page and ensure that your answers to parts of questions are clearly labelled.

1. The origin O is a point on an ocean. The unit vectors $\underline{i}$ is in the direction from O towards east and $\underline{j}$ is the direction from O due north. A ship S is moving with constant velocity $(-12.5\underline{i}+7.5\underline{j})\text{kmh}^{-1}$.

   a) Find the speed and the direction S is moving. Represent its path in a diagram.

   At t hours, the position vector of S is $\underline{s}$ km . When $t=0$, $\underline{s}=40\underline{i}-6\underline{j}$.

   b) Write down $\underline{s}$ in terms of t.

   c) If at a certain time t, the distance of the ship S from the origin is 6 km. Show that t satisfies $at^2+bt+c=0$. Here, the constants a, b and c need to be determined.

   A fixed boat B is at the position with the position vector $(7\underline{i}+12.5\underline{j})$km .

   d) Find the distance of S from B when $t=3$.

   e) Find the time t when

      i. S is due north of B,

      ii. S is due east of B.

2.

a) Relative to a fixed origin O, the points A and B have position vectors $(10\underline{i}+2\underline{j}+3\underline{k})$ and $(8\underline{i}+3\underline{j}+4\underline{k})$ respectively.

i. Find the vector equation of the straight line l, passes through the points A and B.

The point C has position vector $(3\underline{i}+12\underline{j}+3\underline{k})$. The point P lies on l. Given that the vector CP is perpendicular to l.

ii. Find the position vector of the point P

b) Let two straight lines l_1 and l_2 be given by

$l_1 \equiv \underline{r} = (5\underline{i}-\underline{j}+2\underline{k})+\lambda(\underline{i}+\sqrt{15}\,\underline{j}-2d^2\underline{k})$ and

$l_2 \equiv \underline{r} = (7\underline{i}+\underline{j}-4\underline{k})+\mu(\underline{i}+\sqrt{15}\,\underline{j}+d\underline{k})$ respectively, where λ and μ are parameters and d is a real number.

i. Find the values of d, if l_1 and l_2 do not meet each other.

ii. Find the value of d, if l_1 and l_2 are perpendicular to each other.

c) Let $\underline{a}$, $\underline{b}$ and $\underline{c}$ be three non-coplanar vectors. Are the three vectors given by $\underline{f}=5\underline{a}+6\underline{b}+7\underline{c}$, $\underline{g}=7\underline{a}-8\underline{b}+9\underline{c}$ and $\underline{h}=3\underline{a}+20\underline{b}+5\underline{c}$ linearly independent? Justify your answer.

3.

a) The distance from the origin to the plane through point $A(2,p,1)$ normal to the vector $2\underline{i}-3\underline{j}+6\underline{k}$ is 4. Obtain the Cartesian equation of this plane in terms of p and hence find the value of p.

b) Let P_1 and P_2 be two planes, perpendicular to each other given by $P_1 \equiv 3x-ay+2z=0$ and $P_2 \equiv bx+6y-5z=0$ respectively, where a and b are real numbers.

i. Show that $3b-6a=10$.

A straight line l given by the equation $\underline{r} = (4\underline{i} - \underline{j} + 2\underline{k}) + \lambda(2\underline{i} + \underline{j} - \underline{k})$ meets the plane P_1. The angle between the straight line l and the plane P_1 is $\frac{\pi}{6}$.

ii. Find the exact value of a and the value of b in surds form.

4.

a) Let the vector valued functions $\underline{F}$, $\underline{H}$ and $\underline{G}$ be given by

$\underline{F}(t) = 2t\underline{i} - 5\underline{j} + t^2\underline{k}$, $\underline{G}(t) = (1-t)\underline{i} + \left(\frac{1}{t}\right)\underline{k}$ and $\underline{H}(t) = (\sin t)\underline{i} - e^t\underline{j}$.

Determine a function A such that $A(t)e^t = \underline{H}(t)\cdot[\underline{G}(t)\times\underline{F}(t)]$.

b) Find the domain of the vector valued function

$\underline{F}(t) = \left(\frac{1}{t^2-1}\right)\underline{i} + \ln(2-t)\underline{j} + \sqrt{1-\frac{t}{3}}\underline{k}$.

c) The position vector of a particle moving in space at time t is given by

$e^{-t}\tan^{-1}(t)\underline{i} + \left(\frac{1-2t}{3-t}\right)\underline{j} + t\left(\sin\frac{1}{t}\right)\underline{k}$. Find the position vector of this particle as $t \to \infty$.

5.

a) Find the derivative with respect to t, of the function given by $\underline{F}(t)\times\underline{G}(t)$ where $\underline{F}(t) = e^t\underline{i} + t^3\underline{j} + \underline{k}$ and $\underline{G}(t) = t^2\underline{i} + \sin t\,\underline{j} + e^t\underline{k}$.

b) The position vectors of two particles A and B at time t are given by $r_1(t) = e^t\underline{i} + e^{2t}\underline{j} + e^{-t}\underline{k}$ and $r_2(t) = e^t\underline{i} + e^{-t}\underline{j} + e^{at}\underline{k}$ respectively, where a is a real valued parameter. When $t = \ln 2$, the speeds of the two particles A and B are equal. Show that $a = 2^{3-a}$. Also, find the value of a.

c) Find the vector equation of the circle in the plane with center $C(1,2,3)$, radius 6 units and having the perpendicular vectors $\underline{u} = 2\underline{i} - 3\underline{j} - 4\underline{k}$ and $\underline{v} = 12\underline{i} + 4\underline{j} + 3\underline{k}$.

6.

a) Let $\underline{r}(t) = t^2\,\underline{i} + 2t\,\underline{j} + 2t^3\,\underline{k}$. Evaluate $\int_0^1 \underline{r}(t) \cdot \frac{dr}{dt}\, dt$.

b) The position vector of a particle P at time t is given by $e^{-t}\,\underline{i} + \underline{j} - t^2\,\underline{k}$. Let $\underline{F}(t)$ be the variable force on P and it is given by $\underline{F}(t) = e^t\,\underline{i} + t^2\,\underline{j} - 2t\,\underline{k}$. Find the work done by this force from time $t = 0$ to $t = 2$.

c) The acceleration of a particle at time t is given by $\underline{a}(t) = t^2\,\underline{i} - t\,\underline{j} + \underline{k}$. At time $t = 1$, the position vector of the particle was $\underline{j}$ and it was moving with a velocity of $\underline{i} + \underline{j} + \underline{k}$. Find the position of the particle at time t.

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