The Open University of Sri Lanka

Credit Certificates for Foundation Courses in Science

Final Examination– 2018/2019

MAF1501 – Mathematics 1 – Paper I

Duration: Three (03) hours

Date - Saturday, 22nd June 2019 **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least one question from Part B.

Part A – Algebra

1. (a) Find the domain and the range of each of the following functions.
   i. $f(x) = x^2 - 1$
   ii. $f(x) = \frac{1}{x^2-4}$
   iii. $f(x) = \frac{1}{\sqrt{x^2-9}}$

   (b) Show that if $f(x) = \frac{x+2}{x-2}$; $g(x) = \frac{1+x^2}{x}$ and $h(x) = \left(\frac{1-x}{1+x}\right)^2$ then

   $f[g(x)] = \frac{1}{h(x)}$

2. (a) Solve the following index equation.
   i. $2^{2x+1} - 8(2^x) = 0$
   ii. $49^x = \frac{1}{343}$
   iii. $8^{4-x} = 4^{x-3}$

   (b) Solve the following logarithmic equation
   i. $\log_2 4 + 2\log_2 x - \log_2(2x-1) = 2$
   ii. $\log_3 x - 3\log_x 3 + 2 = 0$

3. (a) Solve the following simultaneous equations for x and y.

$xy = 270$ and

$\log_{10} x - 2\log_{10} y = 1$

(b) Prove that $\log_a b = \frac{1}{\log_b a}$. **Hence** show that

$$\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ac + 1} + \frac{1}{\log_c ab + 1} = 1$$

4. (a) Sketch the graph of $y = x^2 + x - 2$ and find
   i. The range
   ii. The axis of symmetry
   iii. The coordinate of the vertex

(b) Let $f(x) = x^2 + 2x + 9; \quad x \in \mathbb{R}$

   i. Find the value of a real constant k, for which the equation $f(x) = k$ has exactly one real root for x.
   ii. Determine the set of values of real constant λ, for which the equation $f(x) = \lambda x$ has no real solution for x.

5. (a) The roots of the quadratic equation $3x^2 + 5x + 6 = 0$ are α and β.

Calculate the values of $\frac{1}{\alpha} + \frac{1}{\beta}$ and $\frac{1}{\alpha\beta}$. Hence write down the equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

(b) Using the Principle of Mathematical Induction, prove that $6^n - 5n + 4$ is divisible by 5 for all positive integers n.

6. (a) i. When the polynomial $x^3 + 2x^2 - px + 1$ is divided by $(x - 1)$ the remainder is 5. Find the value of p.

ii. Find the remainder when $2x^3 + x^2 - 9x + 8$ is divided by $(x - 1)(x + 3)$.

(b) Use the factor theorem to find the real roots of the following polynomials.

i. $x^3 - 3x^2 - 9x - 5$
ii. $x^4 - 2x^3 + 2x - 1$

Part B – Coordinate Geometry

7. The vertices of a triangle PQR are (5,4), (2,2), (-6,2) respectively.
   i. Find the gradients of the sides of the triangle.
   ii. Determine whether the triangle is right angled.
   iii. Calculate the area of the triangle PQR.
   iv. If PQRS is a rectangle, find the coordinates of S.

8. (a) A, B, C are the points (2,1), (6,3) and (8,1) respectively. Find
   i. The mid points of AB and BC.
   ii. The gradients of the lines AB and BC.
   iii. The equation of the perpendicular bisectors of the lines AB and BC.

   and

   iv. Show that the point of intersection of these bisectors is on the x axis.

   (b) i. Find the centre and the radius of the circle given by

   $$x^2 + y^2 - 4x - 6y + 11 = 0$$

   ii. If one end of a diameter of the above circle is (3,4) then calculate the coordinate of the other end of the diameter.

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MAF1501 – Mathematics 1 – Paper II

Duration: Three (03) hours

Date - 23rd June 2019 **Time: 1.30 pm -4.30 pm**

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Answer six (06) questions including at least two questions from each Part.

Part A – Trigonometry

1. (a) The diameter of a wheel in a train is $0.5m$. If the wheel rotates at $3\ r/sec$ calculate the total distance travel by the train in 30 minutes.

   (b) A side of a rhombus has a length of $8\ cm$ and its internal angles are 60^0 and 120^0. Find the length of its diagonals.

   (c) Prove the following identity.
   $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

2. (a) Determine the real constants a and b such that
   $\cos^4\theta + \sin^4\theta = a - b\sin^2(2\theta)$ (Hint: use $\cos^2\theta + \sin^2\theta = 1$).

   (b) Given that $\sin x = \frac{3}{5}$, $\cos y = \frac{5}{13}$ and x and y are acute angles. Without using tables or calculations, find the values of $\cos x$, $\sin y$, $\sin(x+y)$, $\cos(x-y)$ and $\tan(x+y)$.

(c) For all real x, obtain $\sin 3x$ in terms of $\sin x$, using Sin addition formula.

(d) Express the $\sin\theta$ as a rational function of $t = \tan\frac{\theta}{2}$.

3. (a) If $x = \sin\theta - \cos\theta$ and $y = \tan\theta + \cot\theta$ where θ is a real number not equal to integer multiple of $\frac{\pi}{2}$, obtain $\sin 2\theta$

   (i) in terms of x only,

   (ii) in terms of y only.

   Hence obtain a relationship between x and y.

   (b) Using the expansions of $\sin(A - B)$ and $\cos(A - B)$, show that

   $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

   Show that $\tan x = \frac{1-\cos 2x}{\sin 2x}$ for $0 < x < \frac{\pi}{2}$ and deduce that

   $$\tan\frac{\pi}{24} = \sqrt{6} - \sqrt{3} + \sqrt{2} - 2$$

Part B – Statics

4. (a) When two forces P and Q act an angle θ their resultant is R and the angle between R and P is α.

   (i) If $P = 24N$, $Q = 7N$, $\theta = 90^0$ find R and α.

   (ii) If $P = 2N$, $\theta = \frac{\pi}{3}$, $\alpha = \frac{\pi}{6}$ find R and Q.

   (b) The system of coplanar forces $10N$, $2\sqrt{3}N$, $4N$, $8N$ and $4\sqrt{3}N$ are acting on a point O. The first force is acting horizontally while other forces inclined at 30^0, 60^0, 120^0 and 210^0 to the first force.

   (i) Find the algebraic sum of the horizontal and vertical component of this system.

   (ii) Find also the magnitude of the resultant and angle it makes with the horizontal.

5. Forces of magnitude $4p$, $5p$, $6p$ Newtons act respectively along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CA}$ where ABC is a triangle, right-angled at A, with $AB = 4a$ meters and $AC = 3a$ meters. Calculate the magnitude and direction of the resultant of this system of forces and find the point where its line of action meets AB (produced if necessary). A couple of moment M in the plane ABC, is now added to the system so that the line of action of the resultant of the new system passes through A. Find the value of M and the sense of the couple.

6. In the parallelogram $OACB$ with $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$, the point M, is the mid point of AC, OM and AB meets at point P.

   (i) Express the vectors $\overrightarrow{AC}$, $\overrightarrow{AM}$, $\overrightarrow{OM}$ and $\overrightarrow{BA}$ in terms of $\underline{a}$ and $\underline{b}$.

   (ii) When $\overrightarrow{BP}$ is given by $\overrightarrow{BP} = x\overrightarrow{BA}$, find $\overrightarrow{OP}$ with $\underline{a}$ and $\underline{b}$.

   (iii) If $\overrightarrow{OP} = y\overrightarrow{OM}$, express $\overrightarrow{OP}$ with $\underline{a}$ and $\underline{b}$.

   (iv) Hence show that $x = y = \frac{2}{3}$

Part C – Dynamics

7. A train starts from rest at a station and moves along a straight track. Initially the train moves with constant acceleration $3\ ms^{-2}$ until it is moving with speed $24\ ms^{-1}$. It maintains this speed for 20s and then decelerates at $2\ ms^{-2}$ until it comes to rest at a signal
   Calculate

   (a) the distance from station to the signal;

   (b) the average speed of the train for whole journey.

8. Brakes are applied to a moving train, at time $t = 0$, giving it a uniform retardation. At $t = 20\ s$ and at $t = 50\ s$, its displacements from the position of applying brakes are observed to be $750m$ and $1500m$ respectively. Sketch a velocity time graph for the motion of the train, until it comes to rest. Find

   (i) the retardation of the train;
   (ii) the velocity of the train at $t = 50\ s$;
   (iii) the value of t, when the train comes to rest;
   (iv) the total distance traveled by the train.

9. A Particle is projected from the point O with velocity 10ms^{-1} at an angle of $\sin^{-1}\frac{3}{5}$ to the horizontal. It strikes the horizontal plane, through O at A. Find the distance OA.
   Show that if another particle is projected from O with the same angle to the horizontal to hit a target at a height 6m above A, then the velocity of the projection must be $10\sqrt{6}\ ms^{-1}$ (take $g = 10\ ms^{-2}$).