The Open University of Sri Lanka

Credit Certificates for Foundation Courses in Science

Final Examination– 2018/2019

MAF1502 – Mathematics 2 – Paper I

Duration: Three (03) hours

Saturday, 28th December 2019 **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including one question from each Part A and B.

Part A – Trigonometry (Answer one question only)

1. (a) Give the general solution for

$\sin x = p$ and $\cos x = p$, where $(-1 \leq p \leq +1)$

$\tan x = p$, where $(p \in \mathbb{R})$

(b) Find the general solution of the following trigonometric equation.

i. $\cos 2\theta + 3\sin\theta = 2$
ii. $\cos 3\theta + \cos\theta = 2\cos 2\theta$
iii. $\tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta \tan 2\theta)$
iv. $\tan^2 2\theta = 3$

2. (a) Express $\sqrt{3}\cos\theta + \sin\theta$ in the form of $R\cos(\theta - \alpha)$, where R and α are real. Hence, find the general solution of the equation $\sqrt{3}\cos\theta + \sin\theta = 1$.

(b) Let $\tan\frac{\theta}{2} = t$, show that $\cos\theta = \frac{1-t^2}{1+t^2}$ and $\sin\theta = \frac{2t}{1+t^2}$. Hence find the general solution of the equation $11\cos\theta + 7\sin\theta = 13$.

Part – B Coordinate Geometry (Answer one question only)

3. (a) Let l be the straight line passing through the points (4, 0) and (0,2) and m be the straight line passing the points (2, 0) and (0, 3). Find the equations of the straight lines l and m. Hence find the equation of the straight line through the origin and the point of intersection of l and m.

   (b) The image of the point (3, 1) on the straight line $x + 2y + a = 0$ is the point $(\frac{3}{5}, b)$, where a and b are constants. Find the values of a and b.

4. (a) Show that the bisector of the acute angle between two straight lines given by $2x - 11y - 10 = 0$ and $10x + 5y - 2 = 0$ is the bisector of the obtuse angle between two straight lines given by $4x - 7y - 8 = 0$ and $8x + y - 4 = 0$.

Part C – Algebra (Answer three questions only)

5. (a) Find the quotient and remainder when $x^4 - 3x + 10$ is divided by $x - 2$.

   (b) Express $\frac{2x^2-1}{(x-1)^3(x+1)}$ in partial fractions.

   (c) Sketch the graph of the rational function $y = \frac{2}{1+x}$.

6. (a) Solve the following inequality

   (i) $\frac{2}{3}x - 3 > 4$ (ii) $4x(x + 1) \geq 3$ (iii) $3x + 10 > 10x - 11$

   (b) Sketch the graph of $y = (x + 1)(5 - x)$ and solve the inequality $(x + 1)(5 - x) \leq 0$. What are the critical values?

   (c) Sketch the graph of

   (i) $y = |x^2 - 2|$ (ii) $y = |2x - 3|$

7. (a) How many multiples of 9 lie between 10 and 500?

   (b) In an arithmetic series 1st term is 25 and 13th term is -11. Find the common difference and 20th term.

   (c) In a geometric series 2nd term is -6. The sum of 1st , 2nd and 3rd term is 7. Write the first three terms of this series (Write down all possibilities).

8. (a) Find the number of different arrangements of the 10 letters which can be made from the letters of the word PHILOSOPHY.

   In how many of these arrangements do the letters H, I, S, Y appear together?

   Also find the number of different selections of 5 letters which can be made from the 10 letters of the word PHILOSOPHY.

9. (a) If the coefficient of x and the coefficient of x^2 in the binomial expansion of $(1 + px)^{12}$, where p is a non-zero constant, are $-q$ and $11q$ respectively, find the values of p and q.

   (b) Consider the expansion of $\left(\frac{7}{6x} - \frac{6x}{7}\right)^{13}$.

   (i) Write down the general term of this expansion.

   (ii) Show that the coefficient of $\frac{1}{x}$ is 2002.

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Credit Certificates for Foundation Courses in Science

Final Examination– 2018/2019

MAF1502 – Mathematics 2 – Paper II

Duration : Three (03) hours

Date: 29th December 2019 **Time : 1.30 p.m. – 0430p.m.**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including one questions from Part A and two questions from each Part B and Part C .

Part A – Trigonometry

01. (a) Prove that

i. $Cos(A+B)Cos(A-B) = Cos^2A - Sin^2B$

ii. $4Cos\theta Cos\left(\frac{2\pi}{3}+\theta\right)Cos\left(\frac{2\pi}{3}-\theta\right) = Cos3\theta$

(b) Solve the equation for x

$tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

02. (a) Prove that $2cos\frac{\pi}{13} \cdot cos\frac{9\pi}{13} + cos\frac{\pi}{13} \cdot cos\frac{4\pi}{13} = 0$

(b) Sketch the following graphs

i. $y = Sin2x$

ii. $y = Sin2x + \frac{1}{2}$

(c) Prove that $tan^{-1}a - tan^{-1}b = tan^{-1}\left(\frac{a-b}{1+ab}\right)$

Part B – Statics

03. (a) A uniform rod AB of weight W and length $2a$ is kept in equilibrium with the end A in contact with a rough vertical wall; it is supported by a light inextensible string of equal length $2a$ connecting the other end B to a point C of the wall, vertically above A. The rod is inclined at an angle θ to the upward vertical and it lies in a vertical plane perpendicular to the wall.

Find the tension in the string and show that $\theta \geq Cos^{-1}\left(\frac{\mu}{3}\right)$, when μ is the coefficient of friction.

(b) A uniform solid hemisphere of weight W is placed with its curved surface on a rough plane inclined at an angle $\propto$ to the horizontal. It is in limiting equilibrium with its plane face horizontal, when a small weight w is attached to a point on the circumference of its plane face. Show that if μ is the coefficient of friction, then

$$\mu = \frac{w}{\sqrt{W(W+2w)}} = tan \propto$$

04. (a) A weight W is suspended by two light inextensible strings of length a and b from two points at the same horizontal level which are at a distance $\sqrt{a^2 + b^2}$ apart. When the system is in equilibrium find the tensions in the strings.

(b) Two uniform rods AB and BC are equal in length. The weight of AB is 2w and the weight of BC is w. The rods are smoothly hinged at B and the midpoints of the rods are connected by a light inelastic string. The system stands in equilibrium in a vertical plane with A and C on a smooth horizontal table.

If $A\hat{B}C = 2\theta$, show that the tension of the string is $\frac{3}{2}w\ tan\theta$.

Find the magnitude of the reaction at B and the angle it makes with the horizontal.

05. A thin smooth hemispherical bowl of radius a is fixed with its rim upper most and horizontal. A smooth uniform rod AB weight W and length $2l(> 2a)$ rests with the end A on the inner surface of the bowl and a point C of the rod in contact with the rim. Indicate the forces acting on the rod.

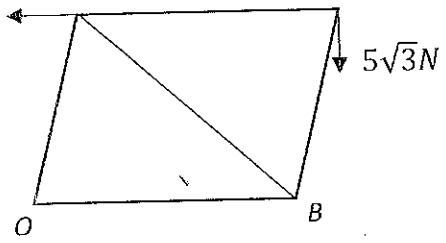
By taking moments about A, show that the reaction R and C is of magnitude $\frac{Wl}{2a}$.

Also, obtain another relation between R and W. Hence, show that the length

CB is $\frac{1}{4}\left(7l - \sqrt{l^2 + 32a^2}\right)$.

06. Five light equal rods *OA, OB, AC,AB* and *BC* are smoothly jointed at their ends to form a framework as shown in the figure.

The framework is smoothly hinged at *O* and carries a weight of $5\sqrt{3}$ newtons at *C*. The framework is held in a vertical plane, with *OB* horizontal by a horizontal force of *P* newtons at *A*. P A C



i. Find the value of *P*
ii. Find the magnitude and the direction of the reaction at *O*.
iii. Using Bow's notation, draw a stress diagram for the framework and find the stresses in all rods, distinguishing between tensions and thrusts.

Part C – Dynamics

07. A motor-boat whose speed is U Km h^{-1} is to intercept a ship which moves with constant velocity $v(< u)Km\,h^{-1}$ in the North – West direction. Initially, the ship is located at a distance d km north of the motor-boat. Draw a velocity triangle and find the direction in which the motor-boat should move in order to intercept the ship. Show that the interception takes place after a time

$$\frac{\sqrt{2}d[\sqrt{2u^2-v^2}+v]}{2(u^2-v^2)} \text{ hours.}$$

08. (a) A particle of mass m slides down *a* smooth face, of inclination $\propto$ to the horizontal, of a wedge of mass M which is free to move on a smooth horizontal table. Show that the acceleration of the wedge is $\frac{mg\,Sin\propto Cos\propto}{M+msin^2\propto}$, and find the reaction between the particle and the wedge.

(b) A train of total mass 300 metric tons is travelling at a constant speed of 54 kilometers per hour, on a straight level track, and the total resistance to the motion is 50 newtons per metric ton. Calculate the power of its engine.

The rear coach, of mass 50 metric tons, then gets disconnected, but the tractive force of the engine is unaltered.

Find,

i. the acceleration of the rest of the train;
ii. the distance moved by the disconnected coach before coming to rest. [Assume that the motion of this coach is retarded by the resistance alone].

09. Two smooth spheres *A, B* of equal radii, are moving, **in opposite directions** on a smooth horizontal table, so as to collide directly. Their masses are 2m, 3m and their speeds are 7u, 3u respectively. The coefficient of restitution between the spheres is e. show that the impulse of the collision is of magnitude *12 mu (1+e).*

If the smaller sphere, *A* is brought to rest b of the impact determine the value of e, and show that $\frac{1}{15}$ of the original kinetic energy is retained in the system.