The Open University of Sri Lanka

Credit Certificates for Foundation Courses in Science

Final Examination– 2018/2019

MAF2501 – Mathematics 3 – Paper I

Duration: Three (03) hours

Date -Saturday, 22nd June 2019 | **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least one question from Part B.

Part A – Calculus

1)

a. Find the following limiting values.

i. $\lim_{x\to 3}\frac{(x+1)-\sqrt{x+13}}{(x-3)}$

ii. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$

iii. $\lim_{\theta\to 0}\frac{\sin^{-1}\theta}{\theta}$ (Hint: use an appropriate substitution)

b. Differentiate the following functions using the first principles.

i. $y = 3x^2 - x + 4$

ii. $y = \frac{1}{x^2}$

2) Differentiate the following functions and simplify.

i. $y = (x^8 + 1)\left(\frac{1}{2}x + 2\right)$

ii. $y = \frac{(2x+1)^2}{(x^2-1)^3}$

iii. $y = \left(x + \sqrt{1 + x^2}\right)^n$

iv. $y = \left(\frac{x+a}{x-a}\right)^5$

3) Differentiate the following trigonometric functions.

i. $y = \frac{\cos x}{\cos x + \sin x}$
ii. $y = \text{cosec}(2x + 1)$
iii. $y = \tan^2(3x)$
iv. $y = \sin^{-1} 3x + \cos^{-1}\left(\frac{x}{2}\right)$
v. $y = \tan^{-1}\sqrt{x}$
vi. $y = \sec^{-1} x$, (use $\sec^{-1} x = 1/\cos^{-1} x$)

4) Differentiate the following functions with respect to x.

i. $y = e^{\sin^{-1} x}$
ii. $y = \frac{1+e^x}{1-e^x}$
iii. $e^y = 1 + x^2$
iv. $y = \ln\left|x + \sqrt{x^2 + a^2}\right|$
v. $y = x^{\sin x}$ (Hint: use the logarithm)

5)

a. The parametric coordinates of a curve are given by, $x = \frac{a(1-t)^2}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ where t is a parameter. Show that $\frac{dy}{dx} = -\frac{b}{a}$.

b. Let $y = (1 + 4x^2)\tan^{-1}(2x)$, show that

i. $(1 + 4x^2)\frac{dy}{dx} - 8xy = 2(1 + 4x^2)$
ii. $(1 + 4x^2)\frac{d^2y}{dx^2} - 8y = 16x$

and

iii. Find $\left(\frac{d^3y}{dx^3}\right)_{x=0}$

6)

a. Find the turning points of the curve $y = x^3 - 3x$ and classify whether they are minimum or maximum.

b. Integrate the following expressions with respect to x.

i. $\int(2x + 5)^6 dx$ $\quad (ii) \int \frac{1}{4+x^2} dx$ $\quad (iii) \int \frac{x^2}{x+1} dx$

$(iv) \int \frac{\sin x}{1 + \cos x} dx$ $\quad (v) \int x\sqrt{x^2 + 1}\, dx$

Part B – Co-ordinate Geometry

7)

a. Obtain the equation of the circle passing through the points of intersection of circles $x^2 + y^2 - 2x - 4y - 4 = 0$ and $x^2 + y^2 + 8x - 4y + 6 = 0$, and the origin.

b. The circle $x^2 + y^2 - 6x + 2y - 17 = 0$ and the line $x - y + 2 = 0$ intersect at points A and B. Find the equation of a circle with AB as the diameter.

8)

a. Show that if the two circles $x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and $x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ intersect orthogonally, then $2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

b. Find the equation of the circle, passes through the points $(2, -1)$ and $(1, -2)$ and intersects orthogonally the circle $x^2 + y^2 - 2x + 3y - 5 = 0$.

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MAF2501 – Mathematics 3 – Paper II

Duration: Three (03) hours

Date: 23rd June 2019 **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least one question from each Part.

Part A – Algebra

1. Using the Principle of Mathematical Induction, prove that

$$\frac{1}{2^2-1}+\frac{1}{3^2-1}+\cdots+\frac{1}{(n+1)^2-1}=\frac{3}{4}-\frac{1}{2(n+1)}-\frac{1}{2(n+2)}$$

for all positive integers n. Deduce that the series $\sum_{r=1}^{\infty}\frac{1}{r(r+2)}$ is convergent and find its sum, hint: $(r+1)^2-1=r(r+2)$.

2.

a) Let $U_r=\frac{3(6r+1)}{(3r-1)^2(3r+2)^2}$ for $r\in\mathbb{Z}^+$ and let $S_n=\sum_{r=1}^{n}U_r$ for $n\in\mathbb{Z}^+$. Find the values of the constants A and B such that $U_r=\frac{A}{(3r-1)^2}+\frac{B}{(3r+2)^2}$ for $r\in\mathbb{Z}^+$. **Hence,** show that $S_n=\frac{1}{4}-\frac{1}{(3n+2)^2}$ for $n\in\mathbb{Z}^+$.

b) Sum of the first n terms of a Geometric Series is given by, $S_n=\frac{a(1-r^n)}{1-r}$ (a-first term, r-common ratio). The second term of Geometric Series is 24 and its sum up to infinity is 100. Find the two possible values of the common ratio and the corresponding first terms.

Part B -Statics

3.

a) i) Write down the position vectors of $r_1 = (3,2,-1)$ and $r_2 = (2,-1,0)$ and find the unit vector in the direction of $(2r_1 - r_2)$.

ii) Show that the points A(3,4,5), B(7,8,9) and C(4,5,6) are lie in a straight line, using vectors.

b) Let ABCD be a trapezium such that $\overrightarrow{DC} = \frac{1}{2}\overrightarrow{AB}$. Also $\overrightarrow{AB} = \underline{p}$ and $\overrightarrow{AD} = \underline{q}$. The point E lies on BC such that $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC}$. The point of intersection F of AE and BD satisfies $\overrightarrow{BF} = \lambda\overrightarrow{BD}$ where λ $(0 < \lambda < 1)$ is a constant. Show that

i. $\overrightarrow{AE} = \frac{5}{6}\underline{p} + \frac{1}{3}\underline{q}$

ii. $\overrightarrow{AF} = (1-\lambda)\underline{P} + \lambda\underline{q}$

Hence, find the value of λ.

4.

a) In the usual notation, let $\underline{i}$ and $\underline{i} + \underline{j}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O. Also, let C be a point on the straight line through A parallel to OB. Show that $\overrightarrow{OC} = (1+\lambda)\underline{i} + \lambda\underline{j}$, where λ is a real number. Find the value of λ such that BC is perpendicular to OB.

b) (i) If $\underline{a} = \underline{i} - 2\underline{j}$ and $\underline{b} = 3\underline{i} + \underline{j}$ are two vectors, find the scalar product of $\underline{a}.\underline{b}$ and the angle between $\underline{a}$ and $\underline{b}$.

(ii) Using scalar product, show that $|a+b|^2 = |a^2| + 2\underline{a}.\underline{b} + |b^2|$

If $\underline{a}$ and $\underline{b}$ are unit vectors with the θ angle between them. Also $\underline{a} + \underline{b}$ is a unit vector derive the angle θ.

5.

a) If position vectors are $\underline{a} = 2\underline{i} + \underline{j} + 3\underline{k}$, $\underline{b} = \underline{i} - 2\underline{j} + \underline{k}$ and $\underline{c} = \underline{i} + 4\underline{j} + 2\underline{k}$, deduced that $\underline{a}.\underline{b} + \underline{a}.\underline{c} = \underline{a}.(\underline{b} + \underline{c})$.

b) The vertices of a triangle ABC are A(3,2,1), B(5,3,-2) and C(2,-3,4). Find the area of the triangle ABC.

6.

a) Show that the diagonals of a rhombus intersect at right angles.

b) In a right-handed cartesian coordinates system, $\underline{i}, \underline{j}, \underline{k}$ are mutually perpendicular unit vectors in the positive direction of ox, oy and oz respectively. If $\underline{a} = 2\underline{i} + \underline{j} - \underline{k}$, $\underline{b} = 3\underline{i} - 2\underline{j} + \underline{k}$ and $\underline{c} = -\underline{i} + 3\underline{j} - 2\underline{k}$ find $\underline{a} \times \underline{b}$. Show that

i. $\underline{c} \times (\underline{a} \times \underline{b}) = -3\underline{i} - 5\underline{j} + 8\underline{k}$ and

ii. $\underline{c}$ and $\underline{a} \times \underline{b}$ vectors, perpendicular to each other.

Part C – Dynamics

7. The ends P and Q, of a light inelastic string are attached into the masses of m and M respectively. The string is passing through the smooth fixed ring of O. M stays freely and m moves in a horizontal circle with a constant speed of u. Show that

i. The slop of OP to the vertical is $\cos^{-1}\frac{m}{M}$

ii. The radius of rotation of m is $\frac{\sqrt{M^2-m^2}}{M}(OP)$

iii. $u^2 = \frac{(M^2-m^2)}{Mm}g(OP)$

iv. The reaction at the ring is $g\sqrt{2M(M+m)}$.

8.

a) A particle with mass is attached to a fixed point in a smooth inelastic string with the length of l. When the particle is moving in a horizontal circle with a constant angular speed, the angle of string to the vertical is $\propto$. Show that the time taken for one rotation is $2\pi\sqrt{\frac{l\cos\alpha}{g}}$.

b) A particle of mass m is attached to one end of a light inelastic string of length l. The other end of the string is attached to a fixed point O and the particle is in equilibrium under gravity. The particle is then projected horizontally with speed u.

   i. Show that the tension in the string when it makes an angle θ with the downward vertical through O is $m\left(3g\cos\theta - 2g + \frac{u^2}{l}\right)$.

   ii. Find the least possible value of u so that the particle can subsequently reach the horizontal level of O.

9. A small smooth particle P of mass m is free to move under gravity in a thin smooth circular tube of radius r and centre O, fixed in a vertical plane. The particle is projected horizontally from the lowest point of the tube with speed $\sqrt{3gr}$.

Explain why the law of conservation of energy can be applied for the motion of the particle.

If v is the speed of the particle when OP makes an angle θ with the downward vertical.

Show that $v^2 = gr(1 + 2\cos\theta)$.

Hence show that the reaction of the tube on the particle changes its direction when $\cos\theta = -\frac{1}{3}$ and find the speed of the particle at that point.