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The Open University of Sri Lanka

Credit Certificates for Foundation Courses in Science

Final Examination– 2018/2019

MAF2502 – Mathematics 4 – Paper I

Duration: Three (03) hours

Saturday, 28th December 2019 **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least one question from each Part.

Part A – Algebra

(1) a) Expand the complex number $(1+i)^6$ and prove that it is a pure imaginary number.

b) Represent the complex numbers $-i, -5+4i, 1+i$ on the Argand diagram by the points A, B, C. Prove that ABC is right angled.

c) Find the Modulus and Argument of (i) $1+i$ (ii) $2\sqrt{3}+2i$.

Hence find the Modulus and Argument of (iii) $(1+i)(2\sqrt{3}+2i)$

(iv) $\frac{2\sqrt{3}+2i}{1-i}$.

Hint: If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ then,

$$z_1 z_2 = r_1 r_2[\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2) \text{ and}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)$$

(2) a) Find the value of θ such that $\frac{3-2i\sin\theta}{1+2i\sin\theta}$ (i) real (ii) purely imaginary.

b) Express the complex number $z = \frac{(1+i)(1+\sqrt{3}i)}{1-i}$ in the form of $r\{\cos\theta + i\sin\theta)$ where $r > 0$ and $0 \leq \theta < 2\pi$.

c) Express z on Argand diagram in the followings (use same plane)

(i) $\text{Arg}(z-1) = \frac{2\pi}{3}$

(ii) $\text{Arg}(z+1) = \frac{\pi}{6}$

Hence find z such that $\arg(z-1) = \frac{2\pi}{3}$ and $\arg(z+1) = \frac{\pi}{6}$.

Part B – Dynamics

(3) A particle is traveling between two points P and Q with linear simple harmonic motion. If the distance PQ is 6m and the maximum acceleration of the particle is 16 ms^{-2}, find the time taken to travel:

a) Distance 1.5m from P,

b) From P to the mid point O of PQ,

c) From the mid point of PO to the mid point of OQ.

(4) A light elastic string of unstretched length l is suspended from a fixed point and a particle of mass m hangs at rest from the free end, the extension being $\frac{3l}{8}$. The particle is pulled down a further distance $\frac{5l}{8}$, and released from rest. Find the modulus of elasticity of the string. Prove that its motion is simple harmonic and find:

(i) The period of the oscillation

(ii) The maximum speed of the particle.

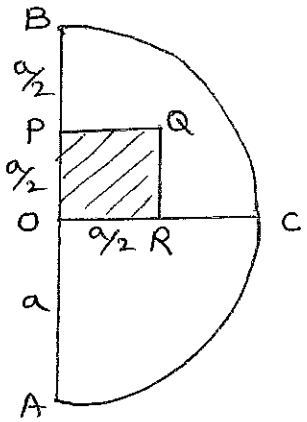
Part C – Statics

(5) A hollow baseless cone of vertex O, semi-vertical angle α and height h is made up of a uniform thin metal sheet of mass σ per unit area. Show that its mass is $\pi\sigma h^2 \sec\alpha \tan\alpha$, and find the position of its centre of mass.

A uniform circular disc, of centre B and radius $h\tan\propto$, made up of the same type of metal sheet is now fixed as the base of the above cone. Show that the distance of the centre of mass of the composite body from O is $\frac{h(\frac{2}{3}\sec\propto+\tan\propto)}{\sec\propto+\tan\propto}$.

The composite body is suspended from a point A of the rim of the base. If AO and AB make equal angles with the downward vertical, show that $\sin\propto=\frac{1}{3}$.

(6) Prove by integration that the centre of gravity of a uniform semicircular lamina of radius a at a distance $\frac{4a}{3\pi}$ from the centre.



ABC is such a lamina with diameter AOB and OC is the radius perpendicular to AB. A square OPQR is cut out from the lamina, P being on OB and OP =a/2. R lies on OC. Find the centre of mass of the remaining part, the distance from OA and OC.

When this lamina is suspended from A and hangs in equilibrium find the angle between AB with the vertical.

Part D – Probability

(7) a) Let $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{2, 3, 4, 5\}$ $B = \{3, 4, 7, 8\}$

$C = \{4, 5, 6, 7\}$

(i) Display above information in a Venn diagram.

(ii) Determine: A' , $A' \cap B'$,$B \cap C'$, $(A \cup B) \cap C$, $(B \cup C) \cap A'$

(iii)Verify that

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C).$$

b) Show that if $p(F) = 0.6, p(G) = 0.25$ and $p(F \cup G) = 0.7$ then F and G are independent events.

c) One die and two coins are tossed at once.

(i) Write down the possible sample space.

(ii) Find the probability that even number in die and at least one head with a coin.

(8) a) Define the probabilities $P(A \cup B)$, $P(A \cap B)$ and $P(A / B)$ in relation to two random events A and B.

b) Two random events A, B have probabilities $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.1$ for the events A, B calculate the probability of

i. Both events occurring

ii. Exactly one of the events occurring and

iii. Neither of the events occurring

c) A batch of 50 LED bulbs, contains 3 defective bulbs. 2 bulbs are selected at random from the batch. What is the probability that:

(i) Both bulbs selected are defective,

(ii) Only one is defective,

(i) Neither is defective?

(9) a) A bag contains 6 red balls and 4 white balls of same size. Three balls are drawn, one at a time, from the bag at random, without replacement. Draw a tree diagram and find the probability that the third ball is red given that the second ball is white.

b) The probabilities that a government servant goes to work by car, bus or train on a certain day are $\frac{1}{10}$, $\frac{2}{5}$ and $\frac{1}{2}$ respectively. The probabilities of his being late for work by these modes of transport are $\frac{1}{5}$, $\frac{1}{2}$ and $\frac{3}{10}$ respectively. If he was late on his particular day, using Bayes' Theorem, calculate the probability that he travelled by train.

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Final Examination– 2018/2019

MAF2502 – Mathematics 4 – Paper II

Duration: Three (03) hours

Date: 29th December 2019 **Time: 1.30 pm -4.30 pm**

You can use calculators. Access to mobile phones during the test period is prohibited.

Answer five (05) questions including at least two (2) question from each Part.

Part A – Calculus

(1) a) Using a suitable substitution evaluate,

i. $\int \frac{xdx}{\sqrt{x+3}}$

ii. $\int \frac{dx}{x^2\sqrt{9-x^2}}$ given that $|x| < 3$

b) Using partial fractions, find $\int \frac{x^2 dx}{(x-1)^3(x+1)}$.

(2) a) By using integration by parts, evaluate the integral $\int x \tan^{-1} x \, dx$

b) Let $I = \int_0^{\pi} e^{-2x} \cos x dx$ and $J = \int_0^{\pi} e^{-2x} \sin x \, dx$.

By using the integration by parts, show that $I = 2J$ and $J = 1 + e^{2\pi} - 2I$.

Hence obtain the value of I and .

c) Evaluate, $\int \frac{dx}{4\cos x + 3\sin x + 5}$.

(3) a) Find $\int cos^4 x \sin^5 x dx$.

b) (i) Find $\int \frac{dx}{\sqrt{3+2x-x^2}}$.

(ii) Find $\frac{d}{dx}(\sqrt{3+2x-x^2})$ and hence find $\int \frac{x-1}{\sqrt{3+2x-x^2}} dx$.

Using the above integrals, find $\int \frac{x+1}{\sqrt{3+2x-x^2}} dx$.

(4) a) Obtain reduction formula for $\int sin^n x dx$.

Hence show that, $\int sin^6 x dx = -\frac{1}{6}\cos x \, sin^5 x + \frac{5}{6}\int sin^4 x dx$.

b) Determine the following definite Integrals.

(i) $\int_1^4 (\sqrt{x} + 2x^{3/2}) dx$ (ii) $\int_0^1 x^2 e^{-x} dx$

c) Find the area of the region enclosed by the straight line $y = 2x$ and the curve $y = x^2$.

Part B - Statistics

(5) a)

Class interval	Frequency
3 - 6	2
6 - 9	5
9 - 12	7
12 - 15	2

For the above data draw,

(i) the frequency polygon,
(ii)the cumulative frequency curve.

b) The following table shows the details of books in a library are categorized according to the no. of pages in a book.

Category	No. of pages	No. of books
A	10-15	5
B	50-90	3
C	90-130	10
D	130-170	2

(i) In this library what category of books can mostly be found, explain.
(ii) What is the average no. of pages in a book.

c) The following data are the amounts spent by a sample of 13 customers in a supermarket on a Monday in rupees.

7, 3, 3, 4, 6, 2, 10, 1, 19, 12, 5, 16, 9

(i) Find the mode
(ii) Group the data into a suitable frequency distribution and calculate the mode.
(iii) Show the mode in a histogram.

(6) a) Find the mean and the standard deviation of the following numbers.

7, 9, 18, 22, 27, 29, 32, 40.

b) The following frequency table describes the age distribution of a randomly selected sample of some citizens in Sri Lanka.

Age	Frequency
0 – 10	37
10 - 20	35
20 - 30	39
30 - 40	46
40 - 50	33
50 - 60	22
60 - 70	20
70 - 80	18

Find the mean and the standard deviation of the data summarized above.

(7) a) Find the median of the following data.

Wages Rs.	No. of labors
60 - 70	5
50 - 60	10
40 - 50	20
30 - 40	5
20 - 30	3

b) Calculate the coefficient of skewness, based on mean median and standard deviation from the following data.

Variable	Frequency
100-110	4
110-120	16
120-130	36
130-140	52
140-150	64
150-160	40
160-170	32
170-180	11

(8) a) Find the median and quartiles from the following data.

Monthly income	No. of persons
Below 50	35
50 – 60	24
60 – 70	21
70 – 80	18
80 – 90	6
90 and above	3

b) From the data relating to two factories A and B given below, find the following:

(i) Mean of the composite set of wages of factories A & b.

(ii) Standard deviation of composite set of standard deviations of wages of factories A & B.

	Factory A	Factory B
No. of workers	100	150
Mean wages	45	55
Standard deviation	7	12

c) Which factory wages are more variable?

Justify your answer (Hint: Find the Coefficient of variation in each factory).