00236

The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2016/2017
Applied Mathematics - Level 04
APU2142/APE4142 Newtonian mechanics I

Duration: Two Hours

Date: 08.01.2018 Time: 01.30pm - 03.30pm

Answer Only Four questions.

1. A particle moving in a straight line, is subject to a retardation of kv^n where v is the speed at time t and n is a positive constant. Find v as a function of t. Show that, if $n < 1$, particle will come to rest at a distance $\dfrac{u^{2-n}}{k(2-n)}$ from the point of projection after a time $\dfrac{u^{1-n}}{k(1-n)}$ where u is the initial speed.
Discuss briefly what happens when
(i) $1 < n < 2$ (ii) $n > 2$.

2. Obtain the radial and transverse components of the velocity and acceleration of a particle moving in a plane in polar co-ordinates.

Masses m_1, m_2 are attached to the ends A and B respectively of a weightless inextensible string AOB and rest on a smooth fixed peg at O, and the portions $OA(=x)$, and $OB(=y)$ of the string are in a straight line. The mass m_1 is now projected horizontally with velocity v perpendicular to OA. If the string remains in contact with the peg, and all the motion takes place in a horizontal plane, prove that the mass m_2 reaches the peg with velocity

$$\frac{v}{x+y}\sqrt{\frac{m_1 y(2x+y)}{(m_1+m_2)}}.$$

3. A particle moves in a plane with velocity v and the tangent to the path of the particle makes an angle ψ with a fixed line in the plane. State the velocity and acceleration components of the particle in intrinsic coordinate. By using these, show that the components of acceleration along the tangent and perpendicular to it are given by

$v\dfrac{dv}{ds}$ and $v^2\dfrac{d\psi}{ds}$ respectively.

A smooth wire in the form of an arc of a cycloid $s = 4a\sin\psi$, is fixed in a vertical plane with the vertex downwards and the tangent at the vertex is horizontal. A small bead of mass m is threaded on the wire and is projected from the vertex with speed $\sqrt{8ag}$. If the resistance of the medium in which the motion takes place is $mv^2/8a$, where v is the speed, then show that the bead comes to instantaneous rest at a cusp $(\psi = \pi/2)$.

4. (a) i. With the usual notation show that the equation of the central orbit is given by $\dfrac{d^2u}{d\theta^2} + u = \dfrac{F}{h^2u^2}$ and $\dot{\theta} = hu^2$.

    ii. At time t the polar coordinates of a particle of unit mass moving in a plane are (r, θ). The only force acting on the particle is $\underline{F} = \dfrac{\mu}{r^3}\underline{e}_r$, where μ is constant and $\underline{e}_r$ is a unit vector along the radial direction and directed away from the pole. Obtain the equations of the possible paths.

   (b) Let the masses of the earth and moon be M_e and M_m respectively and the distance between their centres of mass be d. Show that the distance from the centre of the earth to the point along the line joining of the centres of the earth and moon at which the gravitational attractions due to these two bodies are equal and opposite is $\dfrac{kd}{1+k}$, where $k^2 = \dfrac{M_e}{M_m}$.

5. Derive the equation $\underline{F}(t) = m(t)\dfrac{d\underline{v}}{dt} - \underline{u}\dfrac{dm}{dt}$ for the motion at time t, of a body m of mass $m(t)$ gaining a small amount of mass δm with velocity $\underline{u}$ relative to m, $\underline{F}(t)$ being external force acting on m.

A raindrop falls through a stationary cloud. Its mass m increases by accretion uniformly with the distance x fallen, so that $m = m_0(1 + kx)$, where k is a positive constant. Given that its speed v is zero when $x = 0$, show that

$$v^2 = \frac{2g}{3k}\left[1 + kx - \frac{1}{(1+kx)^2}\right].$$

6. (a) Prove the followings:

    i. The rate of change of total angular momentum $\underline{H}$ about a point O is equal to the total moment of the external forces about O,

    ii. $\underline{H} = \underline{r}_G \wedge M\underline{v}_G + \underline{H}_G$,
    where $\underline{H}_G$ is the angular momentum of the system, $\underline{v}_G$ is the velocity of the centre of mass.

   (b) A uniform circular disk of radius R and mass M is rigidly mounted on one end of a thin light shaft CD of length L. The shaft is normal to the disk at the centre C. The disk rolls on a rough horizontal plane, D, being fixed in this plane by a smooth universal joint. If the centre of the disk rotates without slipping about the vertical through D with constant angular velocity Ω, find the angular velocity, the kinetic energy and the angular momentum of the disc about D.