The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination – 2016/2017
Applied Mathematics – Level 04
APU2144/APE4144 – Applied Linear Algebra and Differential Equations

DURATION: TWO HOURS.

Date: 18.01.2018	**Time: 9.30am-11.30am**

ANSWER ONLY FOUR QUESTIONS.

1. (i) Prove that $\begin{vmatrix} 0 & x & y & z \\ -x & 0 & c & b \\ -y & -c & 0 & a \\ -z & -b & -a & 0 \end{vmatrix} = (ax - by + cz)^2$.

   (ii) If $A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{pmatrix}$ then determine the values of b such that the rank of A is 3.

   (iii) For each case, find the values of a and b for which the system of equations

   $$x + y + z = 6$$
   $$x + 2y + 3z = 10$$
   $$x + 2y + az = b.$$

   has (a) no solution (b) a unique solution (c) infinitely many solutions.

2. (i) Let $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$. Use Cayley-Hamilton theorem to find A^{-1}. Also find the matrix B where $B = A^8 - 5A^7 + 7A^6 - 3A^5 - 5A^3 + 8A^2 - 2A + I$.

   (ii) Transform the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 + 4x_1x_3 - 8x_2x_3$ to a canonical form using orthogonal transformation.

3. Solve each of the following systems of differential equations:

(i) $\dot{x}_1 = x_1 - 2x_2 - 2x_3$

$\dot{x}_2 = 2x_1 + 3x_3$

$\dot{x}_3 = 2x_1 + 3x_2$

(ii) $\dot{x}_1 = -10x_1 + 6x_2 + 10e^{-3t}$

$\dot{x}_2 = -12x_1 + 7x_2 - 18e^{-3t}$

(iii) $\ddot{x}_1 = -10x_1 - 6x_2$

$2\ddot{x}_2 = -12x_1 - 20x_2$

In the above, the dots represent the derivatives with respect to a parameter t.

4. (i) Find a sinusoidal particular solution for the following system of differential equations:

$$4\ddot{x}_1 + \ddot{x}_2 + 3\dot{x}_1 + 3x_1 + 4x_2 = \cos 2t$$
$$\ddot{x}_1 + 4\ddot{x}_2 + 3\dot{x}_2 + 4x_1 + 6x_2 = \sin 2t.$$

(ii) Find the general solution of the differential equation:

$$x^3\frac{d^3y}{dx^3} - 3x^2\frac{d^2y}{dx^2} + 6x\frac{dy}{dx} - 6y = 0.$$

(iii) Find the general solution of the differential equation:

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - \lambda y = 0,\ \text{where } x > 1 \text{ and } \lambda \text{ is a constant.}$$

(Hint: Use $x = \cosh t$).

5. By using the method of characteristic find the general solution of the partial differential equation

$$-2xy\frac{\partial u}{\partial x} + 2x\frac{\partial u}{\partial y} + yu = 8xy.\ (x > 0,\ y > 0)$$

What is the solution which satisfies the condition

$u(x,y) = 2x^{\frac{1}{2}}$ on $y = 0, x > 0$.

6. Use the method of characteristics to solve the equation

$$y^2\frac{\partial^2 u}{\partial x^2} - x^2\frac{\partial^2 u}{\partial y^2} - y^2\left(4x + \frac{1}{x}\right)\frac{\partial u}{\partial x} + x^2\left(4y + \frac{1}{y}\right)\frac{\partial u}{\partial y} = 0$$

in the region $x > 0,\ y > 0$.