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The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2016/2017
Applied Mathematics - Level 03
APU1142/APE3142 – Differential Equations

Duration: - Two hours

Date: 28.12.2017 **Time: 9:30 a.m. – 11:30 a.m.**

Answer **FOUR** questions only.

1.

(a) Find all real values m such that $y = e^{mx}$ is a solution of the differential equation

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 0.$$

(b) Find the integral with respect to x of $f(x) = x^2e^{x^3}$. Hence or otherwise solve the differential equation $\frac{dy}{dx} = e^{x+y} + x^2e^{x^3+y}$.

(c) Solve the Initial Value Problem (IVP) given by

$$(2y+3)\sin^2(x)dy - (y^2+3y+1)\sin(2x)dx = 0, \; y\left(\frac{\pi}{2}\right) = 1.$$

2.

(a) Show that the equation $2x^2\frac{dy}{dx} = x^2 + y^2$ is homogeneous. Using a suitable substitution, solve the above differential equation.

(b) Find the solutions of the simultaneous equations $\frac{\partial u}{\partial x} = x^2$ and $\frac{\partial u}{\partial y} = 3y$.

(c) Solve the differential equation given by $\frac{dy}{dx} = \frac{e^x - xy^2}{x^2y + \sin(y)}$.

3.

(a) Solve the differential equation given by $x\frac{dy}{dx}+2y=x\sqrt{y}$.

(b) Find the particular integral of the differential equation given by

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=2x^2+e^x.$$

4. Show that the power series solution in powers of x, to the differential equation

$\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+4y=0$ can be written as

$y(x)=a_0(1-2x^2)+a_1\left(x+\sum_{k=1}^{\infty}\frac{2^k(-3)(-1)(1)(3)\cdots(2k-3)}{(2k+1)!}x^{2k+1}\right)$ where a_0, a_1 are arbitrary constants and k is a natural number.

5.

(a) Solve the difference equation $u_n+4u_{n-1}+4u_{n-2}=0, n\geq 3$ if $u_1=-2$ and $u_2=12$.

(b) In a certain economy, the *multiplier-accelerated model* with key quantities; Investment (I), Income (Y) and Consumption (C) are linked as

$C_t=\frac{3}{8}Y_{t-1}$ and $I_t=40+\frac{1}{8}(Y_{t-1}-Y_{t-2})$, where the subscript t is the time.

i. Using the equilibrium condition $Y_t=C_t+I_t$, show that

$$Y_t-\frac{1}{2}Y_{t-1}+\frac{1}{8}Y_{t-2}=40.$$

ii. Find the general solution to the homogeneous difference equation in (i) if $Y_0=65$ and $Y_1=64.5$.

6. Let the curve in the xy - plane such that the angle of the tangent line at any point P is three times as big as the angle of inclination of OP where $P \equiv (x, y)$ (see Figure 1).

(a) Using the result $\tan(3A) = \dfrac{3\tan(A) - \tan^3(A)}{1 - \tan^2(A)}$ or otherwise, derive a differential equation connecting $\dfrac{dy}{dx}$, y and x.

(b) Use the substitution $y = vx$ and show that the solution to that differential equation may be written in implicit form as $xy - c(x^2 + y^2)^2 = 0$ where c is an arbitrary constant.

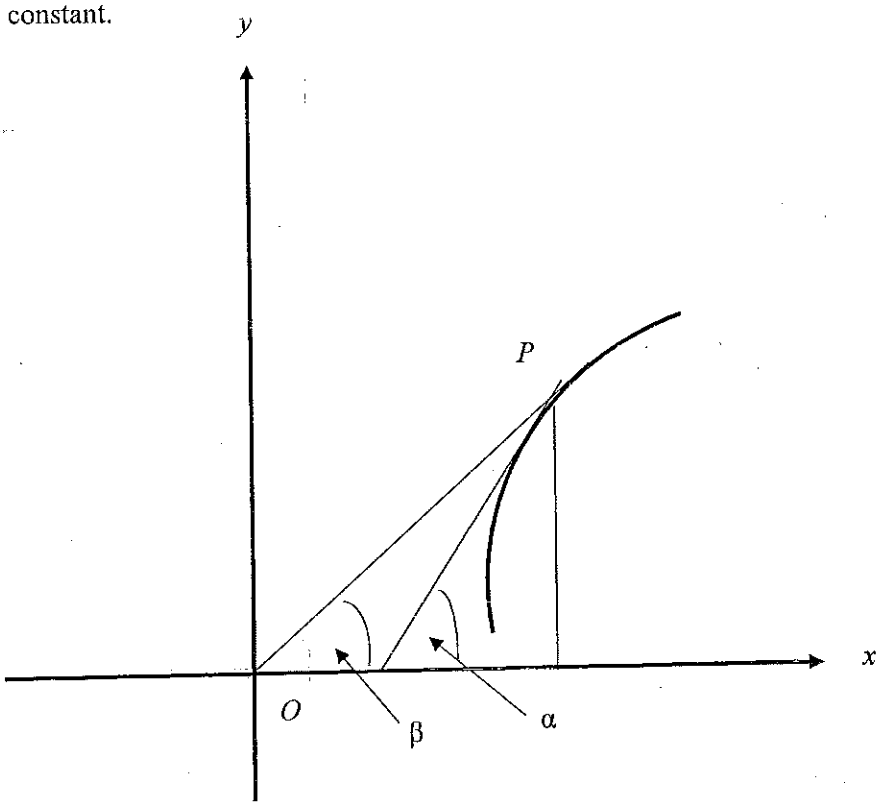

Figure 1

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