00171

The Open University of Sri Lanka
B.Sc.\B.Ed. Degree Programme
Final Examination- 2016/2017
Pure Mathematics - Level 03
PUU1141 \PUE3141– Foundation of Mathematics

Duration: - Two hours

Date: - 03-01-2018 **Time: - 9.30 a.m. – 11.30 a.m.**

Answer **Four** questions only.

1. (a) State, without proof, the De Morgan's Laws for two sets.

(b) Let A and B be arbitrary sets. Without using Venn Diagrams prove the following.

(i) $A\setminus(A\setminus B) = A \cap B$.

(ii) $A \cap B = \emptyset$ if and only if $A \subseteq B^c$.

(iii) $A\setminus (A \cap B) = (A \cup B)\setminus B$.

(c) The relations on the set $\mathbb{Z}$ is defined as follows:

α: xRy if $x > y$.

β: xRy if xy is a square in $\mathbb{Z}$.

γ: xRy if $x + y = 0$.

Determine which of the above relations are reflexive / symmetric / transitive or antisymetric.

2. Prove or disprove each of the following statements.

(i) There are infinitely many $n \in \mathbb{N}$ such that $n^2 + n + 1$ is a multiple of 3.
(ii) There are infinitely many $n \in \mathbb{N}$ such that $n^2 + n + 1$ is not a multiple of 3.
(iii) There are infinitely many $n \in \mathbb{N}$ such that $6n + 5$ is a multiple of 15.
(iv) There are infinitely many $n \in \mathbb{N}$ such that $6n + 5$ is not a multiple of 25.

3. (a) Let x be a rational number such that x^2 is a natural number. Prove that x is an integer.

(b) Prove that for each natural number n, the pair of natural numbers n and $n + 1$ are relatively prime.

(c) Let n be a natural number such that $n \geq 5$. Prove that, if n and $n + 2$ are prime numbers then $n + 1$ is a multiple of 6.

(d) Prove that for each $n \in \mathbb{N}$, such that $n \geq 7$, there exist $l, m \in \mathbb{N}$ such that $n = 2l + 3m$.

4. (a) Let $E = \{2n : n \in \mathbb{N}\}$, the set of even positive integers. Find a bijection from E to $\mathbb{Z}$.

(b) Find a bijection from $(0,1]$ to $[5,6)$.

(c) Let $f(x) = \frac{1+x}{1-x}$, $x \in \mathbb{R}\backslash\{1\}$. Show that f is a bijection from $\mathbb{R}\backslash\{1\}$ to $\mathbb{R}\backslash\{-1\}$.

(d) Let $g(x) = x^2 + 3,\ x \in \mathbb{R}$. Let $A = [0,3)$ and $B = [3,20)$. Find $f^{-1}(A)$ and $f^{-1}(B)$.

5. (a) Prove that if a set A of real numbers has a supremum then supremum is unique.

(b) Let A be a non empty bounded subset of $\mathbb{R}$ such that $inf A \notin A$ or $sup A \notin A$. Prove that the set A has infinitely many elements.

(c) Let $A = [0,1)$, $B = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$, $C = \left\{\frac{n-1}{n} : n \in \mathbb{N}\right\}$.

(i) Prove that each of the above sets is bounded.

(ii) Write down the supremum and infimum of each of the above sets. Prove your answer.

(iii) Does the minimum of $C\backslash B$ exist? Justify your answer.

6. Let x, y and z be positive integers such that $x^2 + y^2 = z^2$. Prove that,

(i) $x^2 + y^2$ is even if and only if z is even.

(ii) xyz is even.

(iii) x is even or y is even.

(iv) xyz is a multiple of 3.