



The Open University of Sri Lanka
 Foundation in Science & Technology
 Final Examination 2017/2018
 PAF2201 – Combined Mathematics I
 Duration:- Three (3) Hours.

Date: 15.09.2017

Time : 9.30 a.m.- 12.30 p.m.

PART A

25 marks for each question

Answer all questions

- 1) Given that $\sin \alpha = \frac{3}{5}$ and $\cos \beta = \frac{9}{41}$, find the values of
 a) $\cos(\alpha + \beta)$ b) $\sin(\alpha - \beta)$

- 2) (i) If $\log_a x = y$, show that $\log_x a = \frac{1}{y}$
 (ii) Solve the equation, $\log_3 x - 3 \log_x 3 + 2 = 0$

- 3) (i) If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$. Find the quadratic equations whose roots are
 (i) $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ (b) α^3 and β^3

- 4) If $x^2 + y^2 = 13$, and $xy = 6$, $x > y > 0$. Find the values for $(x + y)^2$ and $(x - y)^2$. Hence find the values of x and y .

- 5) Find the coefficient of x^{10} in the binomial expansion of $(2x^2 - \frac{3}{x})^{11}$

- 6) Find the modulus and argument of the complex number $(\frac{1-i}{1+i})$.

- 7) Show that two circles $S_1 \equiv x^2 + y^2 - 4x + 2y = 0$ and $S_2 \equiv x^2 + y^2 + 10y + 20 = 0$ touches each other externally and find the contact point of the two circles. Find the equation of the common tangent.

8) Evaluate the limit

$$\lim_{\theta \rightarrow 0} \left(\frac{1 - \cos 2\theta}{\theta^2} \right)$$

9) Differentiate the function with respect to x

$$Y = \sqrt{\frac{1-x^2}{1+x^2}}$$

10) Integrate the function with respect to x

$$\int \frac{x^3 + x + 1}{1+x^2} dx$$

PART B

150 marks for each question in part B

Answer five questions only.

11) a) Prove the following identities

$$(i) \frac{\sin 2\theta + \sin \theta}{1 + \cos \theta + \cos 2\theta} = \tan \theta$$

$$(ii) \sec^2 \theta (1 + \sec 2\theta) = 2\sec 2\theta$$

b) Give the general solutions of $\sin x = p$, $\cos x = p$ and $\tan x = p$ (where $-1 \leq p \leq +1$)

c) Find the general solution of the following equations

$$(i) \sqrt{2} \sec \theta + \tan \theta = 1$$

$$(ii) \tan 2\theta \tan \theta = 1$$

12) a) Let $\lambda \in \mathbb{R}$ and $f(x) = x^2 + 2\lambda x + 2\lambda + 15$, α and β are the roots of the equation $f(x) = 0$.

(i) Find the values of λ such that α and β are real.

(ii) Find the quadratic equation whose roots are $2\alpha + \alpha\beta$, $2\beta + \alpha\beta$ with the terms of λ .

a) When the polynomial function $f(x) = x^3 + ax^2 + bx + 2$ is divided by $(x-1)$ remainder is 4 and when it is divided by $(x+2)$ the remainder is also 4. Find the values of a and b .

13) a) Using the Principle of Mathematical Induction, show that for each positive integer n ,

$$1+4+7+\dots+(3n-2) = \frac{n}{2} (3n-1).$$

b) Find constants A and B such that $\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$ for $r=1,2,\dots$

Hence find $\sum_{r=1}^n \frac{1}{r(r+1)}$ and deduce that

$$\frac{1}{(n+1)(n+2)} + \frac{1}{(n+2)(n+3)} + \dots + \frac{1}{2n(2n+1)} = \frac{n}{(n+1)(2n+1)}$$

14) a) In how many different ways can the letters of the word "THIMBIRIGASYAYA" be arranged when all letters are considered. In how many of these arrangements will all the 'A's be adjacent.

b) Find the greatest term and the greatest coefficient of $(2+3x)^8$.

c) Express each of the following in polar (r, θ) form

(i) $z = 1+i$ (ii) $z = 1-\sqrt{3}i$

15) a) Find the equation of the line which passes through the point of intersection of two straight lines $3x - 5y - 2 = 0$ and $x + y - 4 = 0$ and passes through the point $(-4,3)$.

b) A circle passes through the points P,Q,R which have the coordinates $(3,0)$, $(0,\sqrt{3})$, $(0,-\sqrt{3})$ respectively.

(i) Find the equation of the circle

(ii) Find the coordinate of the centre and the radius of the circle.

(iii) Find the length of the minor arc QR.

16) a) Evaluate the limit

$$\lim_{\theta \rightarrow 0} \left(\frac{\sin 5\theta + \tan 3\theta}{2\theta} \right)$$

b) If $\sqrt{1-x^2} y = \sin^{-1} x$ prove that $(1-x^2) \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} - y = 0$

c) Differentiate with respect to x

(i) $y = \left(\frac{1+2x}{1-3x} \right)^3$

(ii) $y = \ln|x + \sqrt{x^2 + 1}|$

17) a) Evaluate the following indefinite integrals

(i) $\int (x^4 + 3x^3 + \frac{1}{x^2} + \frac{1}{x} + 4) dx$

(ii) $\int \frac{1}{\sqrt{25-x^2}} dx$

(iii) $\int \left(\frac{1-\sin x}{1+\sin x} \right) dx$

b) Evaluate the following definite integrals

(i) $\int_4^9 (2x + 3\sqrt{x}) dx$

(ii) $\int_0^1 x e^{-2x} dx$