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THE OPEN UNIVERSITY OF SRI LANKA

Faculty of Engineering Technology

Department of Mathematics and Philosophy of Engineering

Final Examination (2016/17)

Foundation Course in Science and Technology

PAF 2202 - Combined Mathematics II

Duration: Three (3) hours | **Registration Number: ....**

Date: 22nd October 2017 | **Time: 0930 hours – 1230 hours**

Instructions

- The paper consists of Part-A and Part-B.
- Instructions to answer the questions are given at the beginning of each part.
- Number of pages in the paper is seven (07).
- State clearly any assumptions you required.
- All the symbols are in standard notation unless they are defined.

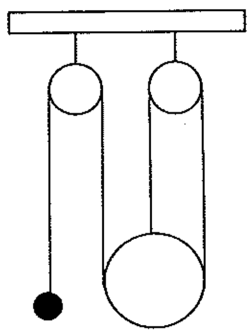
Part-A

- Answer all the questions
- 25 marks are given for each question.

1. A vehicle starts from rest with a uniform acceleration $3f\ ms^{-2}$ on a straight path. After a certain time, the acceleration ceases and it moves with a uniform retardation $2f\ ms^{-2}$ until it comes to rest. The total distance travelled is $a\ m$, and the total time taken is $t\ s$. By sketching the velocity-time graph, show that $t = \sqrt{\frac{5a}{3f}}$.

2. A particle is projected from a point A with a speed of $\sqrt{2ag}\ m\,s^{-1}$. Find the two angles of projection for which it will pass through a point whose height above from A is $\frac{1}{2}a\ m$ whose horizontal distance from A is $a\ m$.

3. As shown in the diagram, one end of an inextensible string which passes over two stationary pulleys and passes under a moveable pulley of mass λm is fixed to a particle of mass m and the other end is fixed to the moveable pulley. If the system is released from rest prove that the tension of the string is $\frac{4\lambda mg}{\lambda + 9}$.



4. A train of mass 10 MT (metric tons) ascends a hill of inclination $\sin^{-1}\left(\frac{1}{100}\right)$ to the horizon. If the frictional force on the train is $15000\,N$ and the engine of the train works at a rate $100\ kw$, then find the maximum speed of the train.

5. Forces of $3P, P, 4P, 2P, 2P, 3P$ Newton act along the sides AB, BC, CD, DE, EF, FA respectively of a regular hexagon $ABCDEF$ of side $2a\,m$. Show that the system is equivalent to a couple and find the couple.

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6. A particle P of mass m is attached to one end of an inextensible string in length l. The other end of the string is fixed to a point A on a ceiling. If P describes a circle of which the center is vertically below A with constant angular velocity ω, then prove that

$$\omega^2 > \frac{g}{l}.$$

7. Two equal uniform beams AB and AC, each of weight W, connected by a smooth hinge at A are placed in a vertical plane with their extremities B and C resting on smooth horizontal plane. They are kept from falling by two strings connecting B and C with the mid points of the opposite beams. If the inclination of each beam to the horizon is $\frac{\pi}{3}$, then show that the tension of each string is $\frac{1}{4}W$.

8. Using the scalar product of vectors, prove that the three altitudes of any triangle are concurrent.

9. A and B are two independent events of a random experiment. If $P(A \cap B) = \frac{3}{25}$ and $P(A \cap B') = \frac{8}{25}$, then find the values of $P(A)$ and $P(B)$.

10. Find the mean and the variance of $1, 2, 3, 4$ and 5. Hence, deduce the mean and the variance of the set that is obtained by transforming the above set using the transformation $y_i = 2x_i + 35$.

Part-B

- **Answer to any five questions.**
- **150 marks are given for each question.**

11. (a) When a motorist is driving with a velocity $6\mathbf{i}+8\mathbf{j}$ the wind appears to come from the direction $\mathbf{i}$. When he doubles his velocity, the wind appears to come from the direction $\mathbf{i}+\mathbf{j}$. Prove that the velocity of the wind is $4\mathbf{i}+8\mathbf{j}$. If the motorist changes his speed but still he drives in the same direction the wind appears to come from the direction $2\mathbf{i}+\mathbf{j}$ find the speed of the motorist.

(b) The horizontal and vertical components of the velocity of an object, projected in a vertical plane are u and λu. If the horizontal range through the projection point is R and it passes a point that has the horizontal and vertical displacements x and y prove that $\lambda x^2 - \lambda Rx + yR = 0$. Deduce that y gets maximum value $\frac{\lambda R}{4}$ when x takes $\frac{R}{2}$

12. (a) A smooth wedge of mass M an angle θ to the horizontal is free to move on a horizontal plane in a direction perpendicular to its edges. A of mass λM $(\lambda \geq 1)$ is projected directly up the face of the wedge with velocity u. Prove that it returns to the point on the wedge from which it was projected after a time $T = \frac{2u(1+\lambda \sin^2 \theta)}{g \sin\theta (1+\lambda)}$. Deduce that $T \geq \frac{4u\sqrt{\lambda}}{g(1+\lambda)}$. Also find the reaction between the wedge and the particle at any time.

(b) Two particles of mass $2m$ and m are connected by a light inelastic string. They are projected simultaneously from the same point on a smooth horizontal table with speeds $2u$ and u respectively, in horizontal directions at right angles. Show that, after the string becomes taut, the both particles move at the angle $\tan^{-1}\left(\frac{6}{7}\right)$ to the direction of the string at the instant tightening. Show also that the loss of kinetic energy due to the tightening of the string is $\frac{5mu^2}{3}$.

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13. Two smooth spheres A, B, of equal radii and masses m, $4m$ respectively, are moving on the surface of smooth horizontal table. The sphere A, moving with speed u and strikes directly the sphere B which is moving in the same direction with speed λu, where $0 < \lambda < 1$. The sphere A is brought to rest by the impact. Find the impulsive force in the collision. Show that, e the coefficient of restitution between A and B, is given by $e = \frac{4\lambda + 1}{4(1-\lambda)}$ deduce that $\lambda \leq \frac{3}{8}$. Given further that 25% of the total kinetic energy is lost in the collision between A and B, prove that $\lambda = \frac{\sqrt{6}-2}{2}$.

14. (a) A horizontal plate oscillates vertically with a simple harmonic motion of period $\frac{2\pi}{n}$ and amplitude a. At time $t = 0$ when the plate at the lowest position a particle of mass m is kept on the plate.

(α) At any time t if the particle will not leave the plate prove that $n^2 \leq \frac{g}{a}$

(β) If $n^2 > \frac{g}{a}$ prove that the particle leaves at $t = \frac{1}{n}\left(\pi - \cos^{-1}\left(\frac{g}{n^2 a}\right)\right)$.

(b) A particle is projected horizontally with a velocity $\sqrt{\frac{1}{2}ag}$ from the highest point of the outside of a fixed smooth sphere of radius a. Show that the particle will leave the sphere at the point whose vertical distance below the point of projection is $\frac{1}{6}a$.

15.(a) A uniform rod AB of length l and weight W is lying on a floor. C is the point on the rod such that $AC = \frac{1}{4}AB$ applying a force at C raises it, which is always at right angles to the rod. Prove that the ratio of the frictional force to the normal contact force with the

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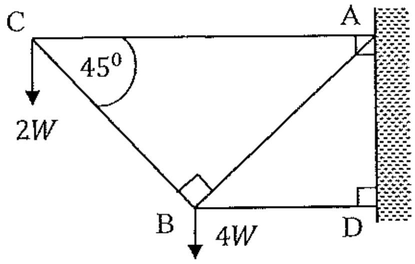
floor is $\dfrac{2\tan\theta}{3\tan^2\theta+1}$ when the rod makes an angle θ. How large must be the coefficient of friction is if the rod is not to slip on the floor while it is being raised?

(b) The framework in figure consists of four light bars AB, BC, AC and DB, freely jointed at B, C, A and attached to a vertical wall at A and D. Weights $2W$ and $4W$ are suspended from C and B. Using Bow's notation find the stresses in all the bars and the reactions at A and D.



16. Find the position the center of gravity of a thin uniform hemispherical shell of radius r. Hence show that the center of gravity of uniform solid hemisphere of radius a is on axis of symmetry at a distance $\dfrac{3a}{8}$ from the center of its base.

A closed vessel consists of thin uniform hemispherical shell and plane circular base made of same uniform material, the radius each being equal to a Show the center of gravity of the vessel is on axis of symmetry at a distance $\dfrac{a}{3}$ from the center of its base.

Such a vessel weight W is completely filed with water weight ω and when suspended from a point of the edge it hangs in equilibrium with the base inclined at an angle θ to the downward vertical. Show that $\dfrac{W}{\omega}=\dfrac{3}{8}\left(\dfrac{3-8\tan\theta}{3\tan\theta-1}\right)$. Hence, deduce that $\dfrac{1}{3}<\tan\theta<\dfrac{3}{8}$.

17. (a) State Bayes' theorem. By examining the chest X – ray, the probability that tuberculosis is detected when a person is actually has the disease, is $0\cdot 99$. The probability that the doctor diagnoses incorrectly that a person has tuberculosis on the basis of an X – ray is

$0 \cdot 001$. In a certain city one in 1000 person suffers from tuberculosis. A person from that town is selected at random and is diagnosed to have tuberculosis. What is the chance that he actually has tuberculosis?

(b) A group of students S_1, S_2, S_3, S_4, S_5 were given the marks for an examination. The mean and the variance of the given marks were 6 and 2 respectively. It is noticed that the boys, S_1 and S_2 have changed their marks. If the others' marks are 8 , 5 , 7 and S_1 was given the least calculate the correct marks of S_1 and S_2. It is proposed to convert the above marks whose the mean and variance are 52 and 18 respectively. If the transformation is $y_i = ax_i + b$ find the values of a , b and calculate the new set of marks.

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