

The Open University of Sri Lanka
 B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
 No Book Test 2019/2020
 Level 03 Pure Mathematics
 PEU3202 Vector Spaces



Duration: - One Hour

Date: - 10-03-2020

Time: 4.15 p.m. to 5.15p.m.

Answer four questions only

1.

(a) Let V be the vector space of all continuous functions $f : [0, a] \rightarrow \mathbb{R}$, where $a > 1$. For arbitrary $p(x), q(x) \in V$ define $\langle p(x), q(x) \rangle = \int_0^1 p(x)q(x)dx$. Show that V is an inner product space.

(b) Let $M = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid a, b, c, d \in \mathbb{R} \right\}$. Note that M is a vector space over the field \mathbb{R} under the usual matrix addition and scalar multiplication.

Let the mapping $T : M \rightarrow M$ be defined by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} a+b & c+d \\ 3c & d \end{bmatrix}$.

- (i) Show that T is a linear transformation,
 (ii) Determine whether the following sets are invariant subspaces of the vector space M over the field \mathbb{R} under T

(I) $W = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \mid a \in \mathbb{R} \right\}$

(II) $W = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \mid a, b, c \in \mathbb{R} \right\}$

2.

(a) Let u and v be any two vectors of a Euclidian Space.

(i) Define the angle between u and v

(ii) Prove that $\|u + v\| \leq \|u\| + \|v\|$

(b) Show that the three vectors $u_1 = (1, 2, 2)$, $u_2 = (1, -1, 2)$ and $u_3 = (1, 0, 1)$ form a basis for E^3 , the usual Euclidean three space. Construct an orthonormal basis for E^3 out of $\{u_1, u_2, u_3\}$ using the Gram-Schmidt process.