

The Open University of Sri Lanka
 Department of Mathematics
 B. Sc/ B. Ed Degree Programme
 No Book Test - 2019/ 2020
 Pure Mathematics- Level 05



PEU5301 - Ring Theory

Duration: One Hour

Date: 23.08.2020

Time: 10.30 a.m. - 11.30 a.m.

ANSWER All Questions

Total Marks = 100

Q1)

Consider the subring of $S = \left\{ \begin{pmatrix} a & 2b \\ b & a \end{pmatrix} : a, b \in \mathbb{Z} \right\}$ of $M_2(\mathbb{Z})$ and the subring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ of \mathbb{R} . Let $\varphi: S \rightarrow \mathbb{Z}[\sqrt{2}]$ be the mapping defined by

$$\varphi \left(\begin{pmatrix} a & 2b \\ b & a \end{pmatrix} \right) = a + b\sqrt{2}.$$

Prove that φ is a ring isomorphism by showing it is:
 (a) a ring homomorphism, (b) one to one, and (c) onto.

30 marks

Q2)

(i) Let R be a commutative ring with unity, and $a \in R$. Prove that $\langle a \rangle = R$ if and only if a is a unit, where $\langle a \rangle = \{b \in R : b = ar \text{ for some } r \in R\}$.

20 marks

(ii) Consider the ideal $\langle 77, 91 \rangle$ of \mathbb{Z} , where $\langle a, b \rangle = \{ax + by : x, y \in \mathbb{Z}\}$. Show that $\langle 77, 91 \rangle$ is a principal ideal of \mathbb{Z} .

20 marks

Q3)

Consider the two polynomials $p(x) = [2]x^2 + [1]$ and $q(x) = [4]x^2 + [5]$ in $\mathbb{Z}_6[x]$.

(a) Compute $p(x) + q(x)$ and $p(x)q(x)$.

20 marks

(b) Determine $\deg(p(x) + q(x))$ and $\deg(p(x)q(x))$.

10 marks

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