

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
No Book Test (NBT) - 2019/2020
Pure Mathematics - Level 05
PEU5303 –Number Theory



Date: 22-08-2020

Time: 10.30 a.m. - 11.30 a.m.

Answer All the questions

Question (01)

- (a) Prove that between any two real numbers there is an irrational number.
- (b) Prove that if $p > 3$ and p is prime, then $p + 4$ or $p + 8$ is composite.
- (c) Prove that an integer of the form $4k + 3$, where $k \in \mathbb{Z}$, cannot be written as sum of two square numbers.
- (d) Prove that any odd positive integer can be written as difference of two square numbers.
- (e) Prove that there are infinitely many $m \in \mathbb{N}$ such that integer of the form $10m + 1$ is divisible by 7.

Question (02)

- (a) Prove that if a is any integer then $360|a^2(a^2 - 1)(a^2 - 4)$.
- (b) Use Fermat's Little Theorem to show that $n^{13} - n$ is divisible by 2730 for all $n \in \mathbb{N}$.
- (c) Find the remainder when $1! + 2! + 3! + \dots + 999! + 1000!$ is divided by 10.
- (d) Compute $2^{372} \pmod{37}$
- (e) Compute $65! \pmod{67}$