The Open University of Sri Lanka
B.Sc. Degree Programme
Applied Mathematics - Level 04
No Book Test-2019/2020
ADU4301/ADE4301 — Newtonian Mechanics I

DURATION: ONE HOUR

Date: 13. 08. 2020   Time: 04.15 p.m. –05.15 p.m.

ANSWER ALL QUESTIONS.

1. A small object P, of mass m_0, is projected vertically upwards from the ground with speed U. As P moves upwards it picks up droplets of moisture from the atmosphere. The droplets are at rest immediately before they are picked up. In a model of the motion, P is modelled as a particle, air resistance is assumed to be negligible and the acceleration due to gravity is assumed to have the constant value of g. When P is at a height x above the ground, the combined mass of P and the moisture is $m_0 (1 + kx)$, where k is a constant, and the speed of P is v.

   (a) Show that, while P is moving upwards $\dfrac{d}{dx}\left(v^2\right) + \dfrac{2kv^2}{(1+kx)} = -2g$

   (b) Show that $v^2 = \dfrac{A}{(1+kx)^2} - \dfrac{2g}{3k}(1+kx)$, where A is an arbitrary constant.

   (c) If $U = \sqrt{2gh}$ and $k = \dfrac{7}{3h}$ then find, in terms of h, the height of P above the ground when P first comes to rest.

2. A uniform rod AB of mass m and length $4a$ is free to rotate in a vertical plane about a horizontal axis through the point O of the rod, where $OA = a$. The rod is slightly disturbed from rest when B is vertically above A.

   (a) Find the magnitude of the angular acceleration of the rod when it is horizontal.

   (b) Find the angular speed of the rod when it is horizontal.

   (c) Calculate the magnitude of the force acting on the rod at O when the rod is horizontal.