00148

**THE OPEN UNIVERSITY OF SRI LANKA**
**BACHELOR OF TECHNOLOGY/ BACHELOR OF SOFTWARE ENGINEERING – LEVEL 05**
**FINAL EXAMINATION – 2014/2015**
**MPZ5140/ MPZ5160 – DISCRETE MATHEMATICS II**
**DURATION: THREE (03) HOURS.**

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**Date: 03rd August 2015** **TIME: 0930hrs – 1230 hrs.**

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**Instruction:**

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each single section.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

## SECTION – A

01. i. Let A be any nonempty set with the operation " $*$ " defined by $x * y = y$. Is the operation,

a) associative? **[10%]**

b) commutative? **[10%]**

ii.

a) Define a group on the set of real numbers with usual notation. **[10%]**

b) Show that $\mathbb{R} - \{0\}$ is a group under ordinary multiplication operation. **[30%]**

iii. Define a semi-group in usual notation. Let " $*$ " be the operation on $\mathbb{Q}$ defined by the following two ways:

c) $x * y = 2(x + y)$

d) $x * y = \dfrac{xy}{5}$

Verify that whether $(\mathbb{Q}, *)$ is a semi- group for each of the above case. **[40%]**

02. i. Let $G = \{i, -i, 1, -1\}$ be a set with complex multiplication operation. Show that $G$ forms abelian group under the operation " $*$ ". (note : $i^2 = -1$) **[40%]**

ii. Let $G_1 = \left\{ \begin{pmatrix} p & q \\ 0 & r \end{pmatrix} \middle| pr \neq 1, p, q, r \in \mathbb{Z} \right\}$. Show that( $G_1, *$ ) is a group, where " $*$ " denotes the usual matrix multiplication. Verify whether $G_1$ is an abelian or not?. **[60%]**

03. i. Define a Homomorphism and Isomorphism for groups in usual notation. **[20%]**

ii. Let $G = \left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} \middle| a \in \mathbb{R} \right\}$, and $G' = \mathbb{R}$. Assuming that $G$ and $G'$ are groups under the usual matrix multiplication.

Let the mapping $\phi : G \to G'$, defined by $\emptyset(A) = a$ where $A = \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ for all $A \in G'$.

Show that $\emptyset$ is both Homomorphism and Isomorphism. **[30%]**

iii. Let $G$ be a group. Show that every element $a$ in $G$ has exactly one inverse in $G$. **[20%]**

iv. If $\phi(e) = e$ and if $\phi$ is a homomorphism of $G$ into $G'$, then show that $\phi(x^{-1}) = [\phi(x)]^{-1} \; \forall x \in G$. **[30%]**

## SECTION – B

04. i. By drawing each of the following graphs, indicate which are simple graphs or not.

a) $G_1 = \{V_1, E_1\}$, where $V_1 = \{a, b, c, d, e, f\}$ and $E_1 = \{\{a,b\}, \{a,d\}, \{a,f\}, \{b,d\}, \{b,f\}, \{b,e\}, \{d,e\}, \{e,f\}\}$. **[10%]**

b) $G_2 = \{V_2, E_2\}$, where $V_2 = \{2,3,4,5,6,7,8,9,10\}$ and two vertices "$s$" and "$t$" are adjacent if and only if $\gcd(s,t) = 1$ where $s, t \epsilon V_2$. **[20%]**

c) $G_3 = \{V_3, E_3\}$, where $V_3 = \{1, 2, 3, \dots, 9, 10\}$, such that "$i$" and "$j$" in $V_3(G_3)$ are adjacent if and only if $i + j$ is a multiple of 4 where $i, j \epsilon V_3$. **[20%]**

ii. Construct a multi-graph of 6 vertex and 7 edges in which every vertex is odd. **[15%]**

iii. Briefly explain connected graph, and disconnected graphs. [10%]

iv Find the number of vertices of a complete graph which has at least 800 edges. [25%]

05. i. What is the largest possible number of verties in a graph with 2160 edges if all vertices have degree at least three? [20%]

ii. Let $G$ be a graph of 16 vertices and 29 edges in which every vertex is of degree 3 or 8. How many vertices of degree 3 and 8 does $G$ have? Construct one such graph $G$. [40%]

iii. Let $G(V, E)$ be a connected graph with at least two vertices and suppose that $|E(G)| < |V(G)|$. Prove that $G$ has at least one vertex of degree one. [40%]

06. i. Define a "Tree graph" and draw a tree with 10 vertices, exactly 6 of which have degree 1 and one of which have degree 4. [20%]

ii. $G$ is the graph whose adjacency matrix $P$ is given by

$$P = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

a) Let $V(G) = \{p_1, p_2, p_3, p_4\}$. Find the number of paths of length four (4) joining vertices $p_2$ and $p_4$. Give the list of paths if any. [50%]

b) Without drawing a diagram of $G$, determine whether $G$ is connected or not. [20%]

a) Draw the graph of adjacency matrix $P$. [10%]

## SECTION – C

07.

i. Iterate the Eco-system growth model for the relation $k_{n+1} - \lambda k_n = 0$ for $k_0 = 0.3$, taking $\lambda = 0.3$ and $\lambda = 1.3$, and draw the diagram. Hence deduce $k_n$ as $n \to \infty$. [40%]

ii. Iterate the Eco-system growth model relationship $y_{n+1} - \lambda y_n - \lambda y_n^2 = 0$, where $\lambda = 2.6$ and $y_0 = 0.3$ (at least 6 iteration steps are necessary)and draw the graph for the relation. [40%]

iii. Explaining the results of (i) and (ii), suggest an appropriate model for the population growth Eco-system. **[20%]**

08. A three dimensional system is governed by the following system of differential equations:

$$\frac{dx}{dt} = -9x - 3y - 7z,$$

$$\frac{dy}{dt} = 3x + y + 3z,$$

$$\frac{dz}{dt} = 11x + 3y + 9z.$$

Where $x, y,$ and $z$ are function of $t$ and at t = 0, $(x, y, z) = (1, 1, 0)$.

Find the phase space value $(x(t), y(t), z(t))$ for $t = 1, 2$. **[100%]**

09. i. Let $L_1 = \{a, ab, c\}$ and $L_2 = \{a^2, ab, b^2\}$ be two languages. Find $L_1L_2$, $L_2L_1$, and $L_1^2$. **[15%]**

ii. Define the term "Grammar".

a) Let $G = \{S, \{0, 1\}, P, S\}$ and $P = \{S \rightarrow 11S, S \rightarrow 0\}$. Find $L(G)$.

b) Construct the production rule to generate each of the following language:

$L_3 = \{a^l b^m c^n : l, m, n \geq 1\}$ and $L_4 = \{(ab)^n : n \geq 1\}$. **[40%]**

iii. Let $M$ be a finite-state machine with state table given below: **[45%]**

| **State** | **Input** | | | **Output** | | |
|---|---|---|---|---|---|---|
| | ***a*** | ***b*** | *c* | ***a*** | ***b*** | *c* |
| $S_0$ | $S_0$ | $S_3$ | $S_2$ | *0* | *1* | *1* |
| $S_1$ | $S_1$ | $S_1$ | $S_3$ | *0* | *0* | *1* |
| $S_2$ | $S_1$ | $S_2$ | $S_3$ | *1* | *1* | *0* |
| $S_3$ | $S_2$ | $S_3$ | $S_0$ | *1* | *0* | *1* |

a) Find the input set, the state set, and the output set.

b) Draw the state diagram of $M$.

c) Starting at $S_0$, what is the output for the input string ***abbccc*** ?