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THE OPEN UNIVERSITY OF SRILANKA
COMMONWEALTH EXECUTIVE MASTER OF BUSINESS/PUBLIC ADMINISTRATION
FINAL EXAMINATION – 2019
MCP1607/MSP9407 – QUANTITATIVE TECHNIQUES
DURATION THREE (03) HOURS

DATE: 31.08.2019 **TIME:9.30 am – 12.30 pm**

INSTRUCTION TO CANDIDATES

a) Answer any five (05) Questions only
b) Each Question Carries 20 Marks
c) Write your index number on every page.
d) Use of Non-programmable calculator is allowed.
e) Necessary statistical tables and formulae given.

Q1. a) Mr. Perera repays an interest free loan of Rs.32,500 by paying Rs.200 in the first month and then increases the payment by Rs.150 every month. How long will he take to clear the loan.

b) Find the sum of the first eight terms of a geometric progression whose first term is five and the common ratio is two.

c) Find the derivative with respect to "x" of the following functions.
(i) $x^2 + 7x + 3$ (ii) $(x^2 + 2)(x^3 + 3)$

d) Find the value of "x" that would minimize the function $x^2 - 8x + 4$

Q2. (a) The monthly sales volume, the unit price and the monthly revenue earned in the Colombo and Kandy Regions for the products "x" , "y", and "z" during June 2019 is given in the table below.

	PRODUCT (QTY)			MONTHLY REVENUE (Rs.)
	X	Y	Z	
COLOMBO SALES	3000	5000	8000	114,000
KANDY SALES	2000	6000	5000	86,000
UNIT PRICE (Rs)	3	5	10	

Develop a matrix equation connecting the above data.

(b) "A" and "B" are two matrices defined as follows.

$$A = \begin{pmatrix} 6 & 8 & 7 \\ 5 & 9 & 4 \\ 7 & 4 & 9 \end{pmatrix} \quad \text{And} \quad B = \begin{pmatrix} 4 & 3 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 8 \end{pmatrix}$$

Evaluate the following

(i) A + B (ii) A - B (iii) 2A + 3B

(c) Use Cramer's rule or inverse metrics method to solve the following simultaneous equations.

$$3x + 2y + z = 10$$

$$x + 2y + z = 8$$

$$x + y + 2z = 9$$

(Q3) (a) A dice is thrown twice and sum of scores calculated. Considering the sum of scores to be a random variable, write down the probability distribution function for the sum of scores.

(b) An Ice Cream vendor sells three flavors namely chocolate, strawberry and vanilla. 50% of sales are chocolate, 30% strawberry and 20% vanilla. In addition ice cream is sold either in cones of cups. The percentage of cone sales for chocolate strawberry and vanilla are 70%, 60% and 40% respectively. A sale is selected at random.

(i) What is the probability that the randomly selected sale was in cone.

(ii) Given that the randomly selected sale was in cone, what is the probability that it is chocolate flavor.

(Q4) (a) (i) Evaluate the expression ${}^{n}C_{r}\, p^{r}\, q^{(n-r)}$
When $n = 5$, $r = 2$, and $p = 0.8$

(ii) Bulbs are sold in boxes and each box contains six bulbs. Probability of any given bulb is damaged is 0.3. What is the probability that exactly two out of the six bulbs are damaged.

(b) (i) Evaluate the expression $e^{-a}\,\frac{a^{x}}{x!}$ When
$a = 2$ and $x = 3$ and $e = 2.71$

(ii) Police observes that on the average there are three accidents per day. What is the probability that there will be two accidents the next day?

(c) The time taken by a candidate to answer a question paper is normally distributed with mean two hours and standard deviation 30 minutes. What is the probability that the candidate will take more than three hours.

(Q5) Data of six machine operators on daily output and years of service is shown in the table below. While "x" represent years of service "y" indicate daily output measured in "000" units. (The total of each column is calculated for your convenience).

OPERATOR NO	x	y	x^2	y^2	xy
1	2	1	4	1	2
2	7	8	49	64	56
3	9	12	81	144	108
4	5	4	25	16	20
5	3	2	9	4	6
6	6	9	36	81	54
TOTAL	**32**	**36**	**204**	**310**	**246**

(i) Calculate the correlation coefficient between daily output and years of service.
(ii) Evaluate the regression equation of the form $y = a + bx$
(iii) Calculate value of output when years of service is 15.
(iv) Calculate the residual for each of the observations in column "y".
(v) Calculate the mean of the residuals and comment on the result.

(Q6) The manager of an organization is concerned about the leave taken by officers. The leave clerk is of the view that the officers with long years of service take more leave. To test this hypothesis, the manager has collected data on leave taken and years of service from 100 randomly selected officers. This data is used to prepare the following contingency table.

YEARS OF SERVICE	LEAVE TAKEN (DAYS)			Total
	Less than 20	20 - 40	More than 40	
Less than 5	15	13	7	35
5 – 10	5	10	10	25
More than 10	5	7	28	40
TOTAL	**25**	**30**	**45**	**100**

Carry out a "Chi-Squared" test to verify the hypothesis that say "leave taken is not related to years of service". Use 5% level of significance.

(Q7) (a) Why do people resort to sampling instead of studying the full population?

(b) Discuss the importance of evaluating sampling error in a study based on a sample.

(c) In a study to estimate the household monthly consumption of electricity, a sample of 400 households where surveyed. It was observed that the mean consumption of electricity was 310 units and the standard deviation was 80 units.

(i) Develop a 95% confidence interval estimate for the monthly consumption of electricity.

(ii) By how much should the sample size be increased to reduce the width of the confidence interval by half?

(Q8) Write short notes on the following topics.

(i) Time Series
(ii) Residual Analysis
(iii) Inferential Statistics
(iv) Corrlation

MATHEMATICAL FORMULAE

CORRELATION COEFFICENT = $$r = \frac{\sum xy - \frac{(\sum x)(\sum y)}{n}}{\sqrt{\left(\sum x^2 - \frac{(\sum x)^2}{n}\right) \times \left(\sum y^2 - \frac{(\sum y)^2}{n}\right)}}$$

LINE OF REGRESSION $y = a + bx$

$$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2}$$

$$a = \frac{\sum y}{n} - b\frac{\sum x}{n}$$

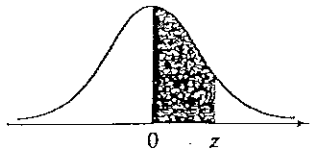
95% Confidence interval = $Mean \pm 1.96\left(\frac{SD}{\sqrt{n}}\right)$

Arithmetic Progression = $S_n = \frac{n}{2}\{2a + (n-1)d\}$

Geometric Progression = $S_n = \frac{a(r^n - 1)}{r - 1}$

Chi- Square Statistic = $\sum \frac{(O-E)^2}{E}$

Normal Curve Areas

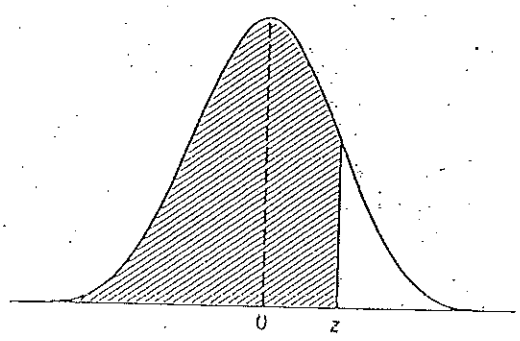


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Source: Abridged from Table I of A. Hald, *Statistical Tables and Formulas* (New York: John Wiley & Sons, Inc.), 1952. Reproduced by permission of A. Hald and the publisher.

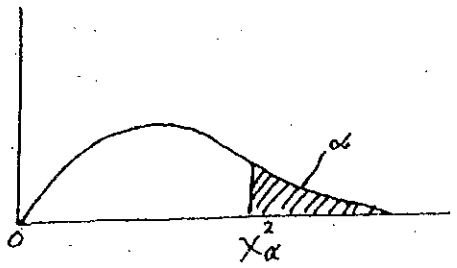
Table A2. Values of z, the standard normal variable, from 0·0 by steps of 0·01 to 3·9, showing the cumulative probability up to z.
(Probability correct to 4 decimal places).

z	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·0	·5000	·5040	·5080	·5120	·5160	·5199	·5239	·5279	·5319	·5359
·1	·5398	·5438	·5478	·5517	·5557	·5596	·5636	·5675	·5714	·5753
·2	·5793	·5832	·5871	·5910	·5948	·5987	·6026	·6064	·6103	·6141
·3	·6179	·6217	·6255	·6293	·6331	·6368	·6406	·6443	·6480	·6517
·4	·6554	·6591	·6628	·6664	·6700	·6736	·6772	·6808	·6844	·6879
·5	·6915	·6950	·6985	·7019	·7054	·7088	·7123	·7157	·7190	·7224
·6	·7257	·7291	·7324	·7357	·7389	·7422	·7454	·7486	·7517	·7549
·7	·7580	·7611	·7642	·7673	·7704	·7734	·7764	·7794	·7823	·7852
·8	·7881	·7910	·7939	·7967	·7995	·8023	·8051	·8078	·8106	·8133
·9	·8159	·8186	·8212	·8238	·8264	·8289	·8315	·8340	·8365	·8389
1·0	·8413	·8438	·8461	·8485	·8508	·8531	·8554	·8577	·8599	·8621
·1	·8643	·8665	·8686	·8708	·8729	·8749	·8770	·8790	·8810	·8830
·2	·8849	·8869	·8888	·8907	·8925	·8944	·8962	·8980	·8997	·9015
·3	·9032	·9049	·9066	·9082	·9099	·9115	·9131	·9147	·9162	·9177
·4	·9192	·9207	·9222	·9236	·9251	·9265	·9279	·9292	·9306	·9319
·5	·9332	·9345	·9357	·9370	·9382	·9394	·9406	·9418	·9429	·9441
·6	·9452	·9463	·9474	·9484	·9495	·9505	·9515	·9525	·9535	·9545
·7	·9554	·9564	·9573	·9582	·9591	·9599	·9608	·9616	·9625	·9633
·8	·9641	·9649	·9656	·9664	·9671	·9678	·9686	·9693	·9699	·9706
·9	·9713	·9719	·9726	·9732	·9738	·9744	·9750	·9756	·9761	·9767
2·0	·9772	·9778	·9783	·9788	·9793	·9798	·9803	·9808	·9812	·9817
·1	·9821	·9826	·9830	·9834	·9838	·9842	·9846	·9850	·9854	·9857
·2	·9861	·9864	·9868	·9871	·9875	·9878	·9881	·9884	·9887	·9890
·3	·9893	·9896	·9898	·9901	·9904	·9906	·9909	·9911	·9913	·9916
·4	·9918	·9920	·9922	·9925	·9927	·9929	·9931	·9932	·9934	·9936
·5	·9938	·9940	·9941	·9943	·9945	·9946	·9948	·9949	·9951	·9952
·6	·9953	·9955	·9956	·9957	·9959	·9960	·9961	·9962	·9963	·9964
·7	·9965	·9966	·9967	·9968	·9969	·9970	·9971	·9972	·9973	·9974
·8	·9974	·9975	·9976	·9977	·9977	·9978	·9979	·9979	·9980	·9981
·9	·9981	·9982	·9982	·9983	·9984	·9984	·9985	·9985	·9986	·9986
3·0	·9987	·9987	·9987	·9988	·9988	·9989	·9989	·9989	·9990	·9990
·1	·9990	·9991	·9991	·9991	·9992	·9992	·9992	·9992	·9993	·9993
·2	·9993	·9993	·9994	·9994	·9994	·9994	·9994	·9995	·9995	·9995
·3	·9995	·9995	·9995	·9996	·9996	·9996	·9996	·9996	·9996	·9997
·4	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9997	·9998
·5	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998	·9998
·6	·9998	·9998	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·7	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·8	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999	·9999
·9	1·0000									



The curve is $N(0, 1)$, the standard normal variable. The table entry is the shaded area $\Phi(z) = \Pr(Z < z)$. For example, when $z = 1{\cdot}96$ the shaded area is 0·9750. Critical values of the standard normal distribution will be found in the bottom row of Table A3.

Chi-Square Table: Values of X_α^2



$X^2_{.10}$	$X^2_{.05}$	$X^2_{.025}$	$X^2_{.01}$	$X^2_{.005}$	df
2.70554	3.84146	5.02389	6.63490	7.87944	1
4.60517	5.99147	7.37776	9.21034	10.5966	2
6.25139	7.81473	9.34840	11.3449	12.8381	3
7.77944	9.48773	11.1433	13.2767	14.8602	4
9.23635	11.0705	12.8325	15.0863	16.7496	5
10.6446	12.5916	14.4494	16.8119	18.5476	6
12.0170	14.0671	16.0128	18.4753	20.2777	7
13.3616	15.5073	17.5346	20.0902	21.9550	8
14.6837	16.9190	19.0228	21.6660	23.5893	9
15.9871	18.3070	20.4831	23.2093	25.1882	10
17.2750	19.6751	21.9200	24.7250	26.7569	11
18.5494	21.0261	23.3367	26.2170	28.2995	12
19.8119	22.3621	24.7356	27.6883	29.8194	13
21.0642	23.6848	26.1190	29.1413	31.3193	14
22.3072	24.9958	27.4884	30.5779	32.8013	15
23.5418	26.2962	28.8454	31.9999	34.2672	16
24.7690	27.5871	30.1910	33.4087	35.7185	17
25.9894	28.8693	31.5264	34.8053	37.1564	18
27.2036	30.1435	32.8523	36.1908	38.5822	19
28.4120	31.4104	34.1696	37.5662	39.9968	20
29.6151	32.6705	35.4789	38.9321	41.4010	21
30.8133	33.9244	36.7807	40.2894	42.7956	22
32.0069	35.1725	38.0757	41.6384	44.1813	23
33.1963	36.4151	39.3641	42.9798	45.5585	24
34.3816	37.6525	40.6465	44.3141	46.9278	25
35.5631	38.8852	41.9232	45.6417	48.2899	26
36.7412	40.1133	43.1944	46.9630	49.6449	27
37.9159	41.3372	44.4607	48.2782	50.9933	28
39.0875	42.5569	45.7222	49.5879	52.3356	29
40.2560	43.7729	46.9792	50.8922	53.6720	30
51.8050	55.7585	59.3417	63.6907	66.7659	40
63.1671	67.5048	71.4202	76.1539	79.4900	50
74.3970	79.0819	83.2976	88.3794	91.9517	60
85.5271	90.5312	95.0231	100.425	104.215	70
96.5782	101.879	106.629	112.329	116.321	80
107.565	113.145	118.136	124.116	128.299	90
118.498	124.342	129.561	135.807	140.169	100