00181

The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination- 2019/2020
Applied Mathematics - Level 03
ADU3300/APU1140/ADE3300/APE3140 VECTOR ALGEBRA

Duration: Two Hours

Date: 28.12.2019 Time: 01.30 pm - 03.30 pm

INSTRUCTIONS TO CANDIDATES

- This paper consists of **TWO** Sections, Section A and Section B. Section A is compulsory and it consists of **ONE** Structured Essay question. You may answer in the space provided under each part of this question.
- Section B consists of **FIVE** essay type questions and answer only any **THREE** of them.
- Always start to answer each question in a new page and ensure that your answers to parts of questions are clearly labeled.
- At the end of the exam, attach Section A to the answer booklet and handover to the supervisor.
- The total marks for this paper is 100 while Section A carries 25 marks.

Section A *Index No:* ____________

1. Let A, B and C be three points lying on a plane P represented respectively by the position vectors $\underset{\sim}{a} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$, and $\underset{\sim}{c} = -\underset{\sim}{i} + 3\underset{\sim}{j} + 2\underset{\sim}{k}$.

   (a) Find the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   (b) Find the vector equation of the straight line l that passes through points A and B.

   (c) Does the point D with the position vector $\underset{\sim}{d} = 3\underset{\sim}{i} + \underset{\sim}{j}$ lie on l? Justify your answer.

   (d) Show that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are perpendicular.

(e) Find the area of the triangle ABC.

(f) Find the equation of the plane P in the form $\underline{r} = \underline{u} + \alpha\underline{v} + \beta\underline{w}$ where $\alpha, \beta \in \mathbb{R}$ and $\underline{u}$, $\underline{v}$ and $\underline{w}$ are vectors.

(g) Find α and β if the point G with the position vector $-5\underline{i} + 5\underline{j} - 4\underline{k}$ lies on plane P.

Section B
Answer **THREE** Questions only.

2. Let $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ be vectors given by $2\underset{\sim}{i}+3\underset{\sim}{j}+p\underset{\sim}{k}$, $\underset{\sim}{i}+q\underset{\sim}{j}-4\underset{\sim}{k}$ and $\underset{\sim}{i}-2\underset{\sim}{j}+5\underset{\sim}{k}$ respectively, and $p, q \in \mathbb{R}$.

   (a) Find p and q such that $\underset{\sim}{a}$ is perpendicular to $\underset{\sim}{b}+\underset{\sim}{c}$.

   (b) Find p and q such that $\underset{\sim}{a} \times \underset{\sim}{b} = -24\underset{\sim}{i} - 4\underset{\sim}{j} + 5\underset{\sim}{k}$.

   (c) Are the vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ linearly independent when $p = 9$ and $q = -5$? Justify your answer.

   (d) Find the values of q such that $|\underset{\sim}{b}+\underset{\sim}{c}| = 3$.

3. Let two vectors $\underset{\sim}{u} = 2\underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{v} = 3\underset{\sim}{i} + 12\underset{\sim}{j} + 6\underset{\sim}{k}$ lie on a plane P. Let C be any point on the plane P with the position vector $\underset{\sim}{c} = \underset{\sim}{i} - \underset{\sim}{j} + \underset{\sim}{k}$, and M be any variable point on the same plane such that the length of CM is 5 units.

   (a) Show that $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are perpendicular.

   (b) Find the unit vectors of $\underset{\sim}{u}$ and $\underset{\sim}{v}$ given by $\hat{\underset{\sim}{u}}$ and $\hat{\underset{\sim}{v}}$ respectively.

   (c) Find the vector equation of the circle lying on the plane P containing the vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ and having C as its centre and radius CM.

   (d) Let N be another point on the above circle in (c) such that the vector $\overrightarrow{CN}$ makes an angle $\tan^{-1} \frac{3}{4}$ with vector $\underset{\sim}{u}$. Find the position vector of N.

4. The vector functions $\underset{\sim}{F}(t)$ and $\underset{\sim}{G}(t)$ are given respectively by
$\underset{\sim}{F}(t) = e^{t}\underset{\sim}{i} + e^{-t}\underset{\sim}{j} + \frac{1}{1-e^{t}}\underset{\sim}{k}$ and $\underset{\sim}{G}(t) = e^{-t}\underset{\sim}{i} + \frac{9e^{t}}{1+e^{t}}\underset{\sim}{j} + (1-e^{2t})\underset{\sim}{k}$.

   (a) Find the domain of each of $\underset{\sim}{F}(t)$ and $\underset{\sim}{G}(t)$.

   (b) Find the value of t such that $\underset{\sim}{F}(t).\underset{\sim}{G}(t) = 5$.

   (c) Find $\underset{\sim}{F}(t) \times \underset{\sim}{G}(t)$.

   (d) Is $\underset{\sim}{F}(0) \times \underset{\sim}{G}(0)$ defined? Justify your answer.

5. Let P_1 and P_2 be two moving particles in space having position vectors in time given respectively by
$\underline{S}_1(t) = t\underline{i} + (2t-1)\underline{j} + 3t\underline{k}$ and $\underline{S}_2(t) = t^2\underline{i} + (2t^2-3)\underline{j} + \underline{k}$, $t \geq 0$.

   (a) Find the time t when P_1 is positioned $\sqrt{11}$ units from the origin.

   (b) Find the velocity vectors of the each particle P_1 and P_2 given by $\underline{V}_1(t)$ and $\underline{V}_2(t)$ respectively.

   (c) Find the time t when both P_1 and P_2 move with the same speed.

   (d) Find the magnitude of the accelerations of P_1 and P_2.

   (e) Let the variable points A and B represent the positions of the particles P_1 and P_2 at any time $t \geq 0$. Let $l \equiv \underline{r} = 4\underline{i} - 3\underline{j} + \underline{k} + \lambda(\underline{i} + \underline{j} + \underline{k})$ represent a straight line in the same space. Find the time t such that $\overrightarrow{AB}$ is perpendicular to l.

6. (a) Let $\underline{r}(t) = 2t\underline{i} + t^2\underline{j} - t^3\underline{k}$. Evaluate

      i. $\int_1^2 \underline{r}(t).\frac{d\underline{r}}{dt}dt$,

      ii. $\int_1^2 \underline{r}(t) \times \frac{d^2\underline{r}}{dt^2}dt$.

   (b) The acceleration vector of a moving particle at any time $t \geq 0$ is given by $\underline{a}(t) = e^t\underline{i} + e^{-t}\underline{j} + e^t\underline{k}$. The position of this particle when $t = 0$ is $-\underline{i}$ and the initial velocity is $\underline{i} + \underline{j} + 2\underline{k}$.

      i. Find the velocity vector $\underline{V}(t)$ of the particle for any $t \geq 0$.

      ii. Find the position vector $\underline{S}(t)$ of the particle for any $t \geq 0$.