00318

The Open University of Sri Lanka
B.Sc. Degree Programme, Level – 04
Final Examination – 2019/2020
PHU4303 – Mathematical Methods for Physics
Duration: 2 hours

Date: 23rd December 2019 Time: 2.00 p.m. to 4.00 p.m.

**Answer any four (4) questions**
Non-programmable calculators are allowed.

**1.**

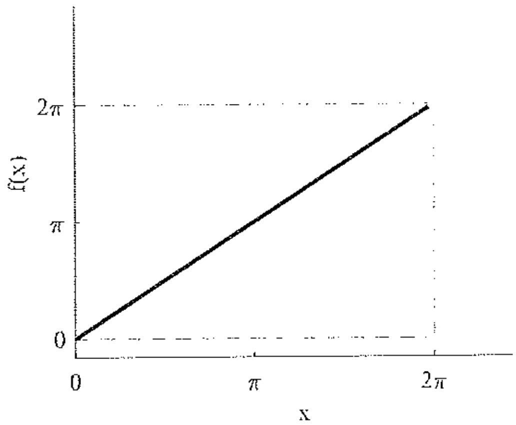
a. Calculate the Fourier series for the function $f(x) = x\, ; 0 \leq x \leq 2\pi$. (Figure 1)

b. Integrate following expressions

i. $\int 3(6y^2 - 1)e^{2y^3-y}.dy$
$Hint: u = 2y^3 - y$

ii. $\int xe^{6x}.dx$

iii. $\int_{-\pi}^{\pi} f(x).dx$ where
$$f(x) = \begin{cases} \sin(x) & x < 0 \\ -2\cos(x) & x \geq 0 \end{cases}$$




**2.**

a. Calculate the Eigen Values of A.

$$A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$$

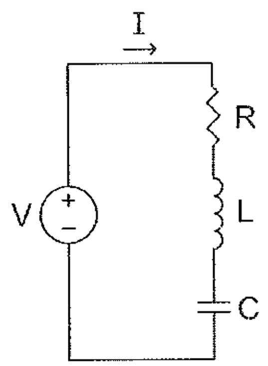
You may use a scientific calculator to solve cubic equations.

b. Following is an RLC circuit.

Impedance of each component is given below.

$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = \frac{1}{i\omega C}$$

R, L and C are constants. $\omega$ is the frequency.



Net Impedance of the circuit is given by $Z_{Net} = Z_R + Z_L + Z_C$

i. Obtain an expression for $Z_{net}$ as a function of $R, L, C$ and $\omega$.

ii. At the resonance frequency, imaginary component of $Z_{Net}$ will be zero, Calculate $\omega$ at resonance.

iii. Calculate $|Z_{Net}|$ at resonance.

**3.**

a. Ray transfer matrices allow calculation of the behavior of light beams. Let

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ \frac{n_2 - n_1}{R n_2} & \frac{n_1}{n_2} \end{bmatrix}, C = \begin{bmatrix} h_1 \\ \theta_1 \end{bmatrix}, D = \begin{bmatrix} h_2 \\ \theta_2 \end{bmatrix}$$

i. If $n_1 = 1.5$ , $n_2 = 1.0$, $R = 0.2$, $h_1 = 0.3$, $\theta_1 = 0.8$, rewrite matrices $A_1$, $A_2$ and C using numerical values.

ii. If $D = A_1 A_2 C$ calculate $h_2, \theta_2$

b. Show that

$$det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = -det \begin{bmatrix} a & b & c \\ g & h & i \\ d & e & f \end{bmatrix}$$

c. A water patch in the rainforest is invaded by an invasive water plant. Total area of the water patch is 120 m$^2$. A botanist one day visits the water patch and discovers 15 m$^2$ of the pond is covered by the invasive plants. He visits the site a week later and discovers it had spread to 30 m$^2$.

i. By assuming that the spread of the invasive plant is arithmetic, calculate how long it will take to completely cover the pond.

ii. By assuming that the spread of the invasive plant is geometric, calculate how long it will take to completely cover the pond.

iii. Is it possible to calculate the date pond was first infected in each case? Explain.

4.

a. A researcher measures the temperature (T) above a certain land mass and found that it can be described by the following equation.

$$T(x,y,z) = 2x + 3y + \frac{xy}{2} - z^2 + 15$$

i. What is the temperature at the point (1,1,2)?

ii. What is the temperature at the point (2,2,3) ?

b.

A bird at the location Q (x,y,z) wish to fly in the direction of the highest temperature drop. Obtain an expression for this direction.

c.

Changes of temperature causes pressure changes which moves air molecules. Velocity of these molecules are described by the following equation.

$$\vec{V} = 2xy\hat{\imath} + 3xy\hat{\jmath} - x^2y^2\hat{k}$$

Obtain an expression for the net inward/outward flux of air molecules from a small volume around the point (x,y,z).

d.

Obtain an expression for the rotation of the air molecules around the point (x,y,z).

5.

A rare disease occurs in 1 in 2 million people in a population. A medical diagnosing test is developed to detect people with the illness. If a person has the disease, test always turns positive (i.e. there are no false negatives) . If a person doesn't have the disease, there is $\frac{1}{10^5}$ probability test might mistakenly turn positive (false positive).

a. In a country with 50 million people, estimate how many people having the disease. Similarly, estimate how many people not having the disease.

b. Government decides to screen everyone using above test. How many people would get a "positive" result ?

c. From the people who get a positive for the test, how many actually have the diseases?

d. A person gets a positive in the test, what is the probability he actually has the disease?

e. If that person did the test again and test still become true, what is the probability he actually has the disease?

6.

a. Work done by an ideal gas during a reversible process is described by the following equation. 'w' is the total work done by the gas.'v' is the volume of the gas.

$$dw = P.dv$$

i. For an isothermal process, $P = \frac{NRT}{V}$ where N,R,T are constants. Show the total work by the change of volume from $V_a \; to \; V_b$ is given by

$$w = nRTln\left(\frac{V_b}{V_a}\right)$$

ii. For an adiabatic process, $PV^{\gamma} = k$ where k is a constant. Show the total work done by the change of volume from $V_a \; to \; V_b$ is given by

$$w = \frac{k\left(V_B^{(1-\gamma)} - V_A^{(1-\gamma)}\right)}{(1-\gamma)}$$

b. **Use bisection method** to find a root of the following polynomial in the given interval.

$$x^2 - x - 2 \; ; -8 \leq x \leq 8$$

i. Fill the following table. You may add or remove rows if needed. Your answer needs to be accurate up to 0.1.

| Iteration | a | b | midpoint | f(midpoint) |
|---|---|---|---|---|
| 1 | | | | |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |

ii. What is your final answer?

-End-