The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Final Examination - 2019/2020

Level 04 Applied Mathematics

ADU4300/ADE4300– Statistical Distribution Theory

Duration: - Two Hours

Date: - 05-01-2020 **Time: 1.30 p.m. to 3.30 p.m.**

Instructions

- This examination is of **Two hours** duration.
- There are two parts to the question paper. **Part A** consists of 10 compulsory multiple choice questions. Twenty five (25) marks are allocated for these questions (2.5 marks for each question).
- **Part B** consists of 5 essay questions. Answer 3 questions out of 5 questions.
- For the **part A** write your answers in the question paper.
- At the end of the examination, handover the **Part A** along with the answers to **Part B.**
- Non programmable calculators are permitted. Statistical tables are provided.

Part A

Registration No: ....

Underline the correct answer.

1. At Ingles Market has been determined that customers arrive at the checkout section according to a Poisson distribution at an average rate of 12 customers per hour. What is the probability that one customer will be arrived at the checkout section in the next hour?

   (a) $\frac{12^1 e^{-12}}{1!}$ (b) $1-\frac{12^0 e^{-12}}{0!}$

   (b) $\frac{12^0 e^{-12}}{0!}$ (d) none of the above

2. For larger values of n > 100, smaller values of p < 0.01 and np < 10, Binomial Distribution

   (a) loses its discreteness

   (b) gives oscillatory values

   (c) approximately tends to Poisson Distribution

   (d) none of the above

3. For the distribution X~Unif(0,5), which of the following is/are true?

   I. $\Pr(X=3) = 0.2$

   II. $\Pr(3 \leq X \leq 7) = 0.4$

   III. $\Pr(X > 2.5) = 0.75$

   (a) I only (b) I and II (c) II only (d) III only

4. $E[X-E(X)]$ is equal to

   (a) $E(X)$ (b) $Var(X)$ (c) 0 (d) $E(X)- X$

5. The height of an adult male is known to be normally distributed with mean of 175 cm and standard deviation 6 cm. What is the value of Q1 in this distribution of heights?

   (a) 0.6745 cm (b) 170.95 cm (c) 179.02 cm (d) 182.34 cm

6. Pulse rates of adult men are approximately normal with a mean of 70 and a standard deviation of 8. Which choice correctly describes the proportion of men that have a pulse rate greater than 62?

   (a) the area to the left of z = -1 under a standard normal curve.

   (b) the area between z = −1 and z = 1 under a standard normal curve.

   (c) the area to the right of z =1 under a standard normal curve.

   (d) the area to the left of z = 1 under a standard normal curve.

7. Which one of the following probabilities is a "cumulative" probability?

   (a) The probability that there are exactly 4 people with Type O+ blood in a sample of 10 people.

   (b) The probability of exactly 3 heads in 6 flips of a coin.

   (c) The probability that the accumulated annual rainfall in a certain city next year, rounded to the nearest inch, will be 18 inches.

   (d) The probability that a randomly selected woman's height is 67 inches or less.

8. Suppose that a quiz consists of 20 True-False questions. Suppose that a student who has not studied for the examination will just randomly guesses the answers to each question. Which choice correctly describes the probability that the student will get 8 or fewer answers correct?

   (a) the probability that X=8, in a binomial distribution with n = 20 and p=0.5.

   (b) the area between 0 and 8 in a uniform distribution that goes from 0 to 20.

   (c) the cumulative probability for 8.5 in a normal distribution with mean =10 and variance = 5.

   (d) None of the above

9. Sum of n independent Exponential random variables with parameter λ results in

(a) Uniform random variable

(b) Binomial random variable

(c) Gamma random variable

(d) Normal random variable

10. The time it takes a student to finish buying his/her text books, W, is a normal random variable. This variable W is the sum of two other normal variables, X and Y ($W = X + Y$), where X = the time to wait in line at the ATM machine to get cash, and Y = the time to wait in line at the cashier to buy the books. Assume that $X \sim N(4,25)$ and $Y \sim N(8,144)$. Also assume that X and Y are independent random variables.

What is the standard deviation for the time it takes a student to finish buying his/her text books?

(a) 5 minutes

(b) 12 minutes

(c) 7 minutes

(d) 13 minutes

Part B

1.

A manufacturer of electronic calculators offers a warranty. If the calculator fails for any reason during this warranty period, it is replaced. The time to failure X is well modeled by the following probability distribution.

$$f(x) = 0.25e^{-0.25x} \quad ; x > 0$$

(i) Find the expected time to failure of a randomly selected calculator.

(ii) If the manufacturer of electronic calculators offers a four years warranty period, what percentages of the calculators will replace?

(iii) Find the cumulative distribution function of the time to failure of a randomly selected calculator.

(iv) Determine the probability that a calculator fails in the interval from 3 to 5 years.

(v) Determine the number of years within which 50% of the calculators in a batch of production have failed.

2.

(a) Some traffic lights have three phases: ***stop 20%*** of the time, ***wait 10% of the time*** and ***go 70%*** of the time. Assuming that you only cross a traffic light when it is in the ***go*** position. Thilak has to pass 10 such traffic lights on his way to school.

- (i) Find the probability that Thilak has to ***wait or stop*** at exactly 3 traffic lights on his way to school.
- (ii) Find the probability that Thilak has to ***wait or stop*** for the second time at the fifth traffic light he meets on his way to school.
- (iii) Find the probability that Thilak cross the second and third traffic lights on his way to school.

(b) Random variable X has the probability mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0,1,2,3,\dots\dots\dots$$

Let $M_x(t)$ be the moment generating function of X.

- (i) Show that $M_x(t) = e^{\lambda(e^t-1)}$
- (ii) Using part (i), show that $E(X) = \lambda$ and $Var(X) = \lambda$

3.

(a) Suppose that X_1, X_2, X_3, X_4 are independent random variables described as

$$X_1 \sim N(3,4) \qquad X_2 \sim N(5,4) \qquad X_3 \sim exp(2) \qquad X_4 \sim gamma(2,2)$$

Find the following probabilities. Show your calculations and state the justifications clearly. You may use the gamma table at the end of the paper wherever necessary.

(i) $\Pr[21 < (2X_1 + 3X_2) < 25\ )]$

(ii) $\Pr[(X_3 + X_4) > 2\ )]$

(iii) $\Pr\left[\ \left(\frac{(X_1-3)}{2}\right)^2 < 3\right]$

(b) Suppose that a sample of $n = 2000$ nails of the same type are obtained at random from an ongoing production process in which 8% of all such nails produced are not on to the required specification. What is the probability that in such a sample 200 or higher nails will be up to the required specification?

4.

The joint distribution of random variables X and Y is given below.

P(X=x,Y=y)		*X*			
		0	**1**	**2**	**3**
Y	**1**	0.12	0.1	0.1	0.06
	2	*0.01*	0.08	*0.00*	*0.05*
	3	*k*	0.1	*0.3*	*0.02*

(i) Find the value of k.

(ii) Calculate the following probabilities.

(I) $Pr(X<1, Y=2)$ (II) $Pr(Y = 3)$ (iii) $Pr(X < 2)$

(iii) Find the marginal distribution of Y.

(iv) A student says that "the random variables X and Y are independent ". State whether the students' statement is true or false. Justify your answer.

(v) Calculate $Pr(X = 0 \mid Y = 2)$

(vi) Calculate $E(X \mid Y = 2)$

5.

(a) In a particular school, Mathematics mark of a randomly selected student in grade ten is denoted by X. From the past experience it is known that $X \sim N(\mu, \sigma^2)$; $\mu = 50$ and $\sigma = 13$.

(i) Find the probability that Mathematics mark of a randomly selected student in grade ten score more than 75.

(ii) Find the proportion of students in grade ten fall in to the range of mathematics marks 40 to 75.

(iii) Suppose that 200 students will be in grade ten in the school in 2020. Find the expected number of students who score less than 40 marks from the students in grade 10 in 2020.

(b) Life time of a light bulb manufactured by ABC Company is normally distributed with mean 600 days and a standard deviation of 100 days. Random sample of 10 bulbs from the above population were tested and sample mean $\bar{X}$ was estimated. Find the probability that sample mean $\bar{X}$ exceeds 550 days.

Left tail values of Standard Gamma Table
W - gamma(α,1)
This table contain the probabilities Pr(W ≤ w)

	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432