THE OPEN UNIVERSITY OF SRI LANKA

B.Sc./B.Ed DEGREE PROGRAMME – LEVEL 04

FINAL EXAMINATION -2019/2020

PURE MATHEMATICS

PEU4302/PEE4302/PUU2142 – Linear Algebra

DURATION: Two (02) Hours

Date: 02.01.2020 **Time:1.30p.m.-3.30p.m.**

Answer FOUR questions only.

1. (a) Prove that a square matrix is invertible if and only if it is non-singular.

   (b) Find the inverse of the following matrix:

$$\begin{pmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{pmatrix}.$$

   (c) (i) Find the values of x so that the rank of $A \leq 2$, where A is the matrix given by

$$\begin{pmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{pmatrix}.$$

   (ii) What is the rank of A for the above x values. Justify your answer.

2. (a) Find non- singular matrices P and Q such that PAQ is normal, where

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{pmatrix}.$$

   Hence, find the rank of A.

(b) Show that the following matrix B satisfies the Cayley-Hamilton Theorem:

$$B = \begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}.$$

Check whether B is invertible. Justify your answer.

(c) Use elementary transformations to reduce the following matrix C to the triangular form and hence find the rank of C.

$$C = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

3. (a) Solve the following system of homogeneous equations:

$$4x + 2y + z + 3w = 0$$
$$6x + 3y + 4z + 7w = 0$$
$$2x + y + w = 0.$$

(b) For what values of η the following system of equations will have

(i) a unique solution,
(ii) no solution,
(iii) infinitely many solutions.

$$x + y + z = 1$$
$$x + 2y + 4z = \eta$$
$$x + 4y + 10z = \eta^2$$

Solve the given system completely in each of the above case.

4. (a) Find the row reduced echelon form of the following matrix:

$$\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{pmatrix}$$

(b) Show that $\begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix} = xyz(x-y)(y-z)(z-x)$.

(c) Solve the following system of linear equations by LU-decomposition:

$$2x+5y-z+t=13$$
$$x-2y+z+2t=5$$
$$3x+3y-2z-t=8$$
$$-x-y+2z-2t=-11.$$

5. (a) Show that any square matrix A can be written as the sum of a symmetric matrix B and skew- symmetric matrix C.

(b) Transform the following quadratic form to a canonical form by an orthogonal transformation:

$$x_1^2+5x_2^2+x_3^2+2x_1x_2+2x_2x_3+6x_3x_1.$$

(c) Determine the nature, index and the signature of the above quadratic form.

6. (a) Let A and B are two square matrices of same order. If A and B are Hermitian, show that $AB-BA$ is Skew-Hermitian.

(b) If $N=\begin{pmatrix} 0 & 1+2i \\ -1+2i & 0 \end{pmatrix}$, show that $(I-N)(I+N)^{-1}$ is a unitary matrix, where I is the identity matrix.

(c) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 2 & -1 \\ 5 & -2 \end{pmatrix}$.