The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme
Final Examination -2019/2020
Applied Mathematics – Level 05
APU3240 – Numerical Methods

Duration: Three Hours

Date: 27. 12. 2019 Time: 02.00 p.m. – 05.00 p.m.

Answer Five Questions Only.

1. (a) Show that recurrence relation to find the root of a given equation by method of False Position is given by $x_{n+1} = x_{n-1} - \dfrac{f(x_{n-1})}{f(x_n) - f(x_{n-1})}(x_n - x_{n-1})$ where $n = 1, 2, 3, \ldots\ldots$

   (b) Show that the equation $x^4 + x^2 - 80 = 0$, has a root in the interval [2, 3] and use method of False Position to find the root correct to four decimal places.

2. (a) Prove that

   (i) $E = (1 - \nabla)^{-1}$, (ii) $\delta = E^{1/2} - E^{-1/2}$,

   (iii) $\mu = \frac{1}{2}\left(E^{1/2} + E^{-1/2}\right)$ where ∇, δ, E and μ are the backward difference, the central difference, the shift and the average operators respectively.

   (b) Derive Gregory- Newton backward interpolation formula.

   (c) The table given below shows the melting point of an alloy of lead and zinc, where y is the temperature in ^0C and x is the percentage of lead in the alloy.

$x\%$	40	50	60	70	80	90
y °C	184	204	226	250	276	304

   Using suitable interpolation formula, find the melting point of the alloy containing 84% of lead.

3. (a) Derive Cubic Spline interpolation formula

$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}(y_{i-1} - 2y_i + y_{i+1})$, for $i = 1$ to $n-1$, where $f''(x_i) = M_i$ and $x_{i+1} - x_i = h$.

(b) Following values of x and y are given.

x	1	2	3	4
y	1	2	5	11

Using Cubic Spline interpolation method evaluate $y(1.5)$.

4. (a) Derive the Simpson's One –Third Rule.

(b) If the interval $[a, b]$ is divided into $2n$ sub intervals and corresponding ordinates are denoted by y_0, $y_2 \ldots\ldots y_{2n}$ then show that the error in Simpson's One –Third rule is given by $|E| < \frac{(b-a)h^4}{180}M$, where M is the numerically greater value of y_0^{iv}, $y_2^{iv} \ldots\ldots y_{2n-2}^{iv}$.

(c) Evaluate $\int_0^1 \frac{x^2}{1+x^3}dx$ using Simpson's One –Third rule with $h = 0.25$. Hence find an approximate value for $\ln 2^{1/3}$.

5. (a) (i) Derive formula for Picard's method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = 2x - y^2$ with the initial condition $y(0) = 0$.

(b) Applying Euler's method with step size $h = 0.1$, find the value of $x(0.5)$ for the initial value problem $\frac{dy}{dx} = \frac{4-x}{x+y}$ subject to the initial condition $y(0) = 1$.

6. (a) Applying Taylor series method of fourth order for the differential equation

$\frac{dy}{dx} = x^2 + y^2$ subject to the initial condition $y(0) = 1$, evaluate $y(0.1)$ and $y(0.2)$.

(b) Applying Taylor series method of third order for the system of differential equations $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$ subject to the initial conditions $y(0) = 2,\ z(0) = 1$ find $y(0.1),\ y(0.2),\ z(0.1)$ and $z(0.2)$.

7. (a) State fourth order Runge-Kutta algorithm to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = x + y$ subject to the initial condition $y(0) = 1$, at $x = 0.1,\ 0.2$ and 0.3.

(c) Solve the second order differential equation $\frac{d^2y}{dx^2} = y^3$ with the initial condition $y(0) = 10$, $y'(0) = 5$ using Runge-Kutta method of fourth order and evaluate $y(0.1)$.

8. (a) State Adam-Bashforth Predictor – Corrector Method to solve $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Applying Taylor series method for $\frac{dy}{dx} = \frac{1}{2}(x + y)$ with the initial condition $y(0) = 2$,

find $y(0.5),\ y(1)$ and $y(1.5)$ correct to four decimal places. Hence find $y(2)$ using Adam-Bashforth Predictor – Corrector Method.