The Open University of Sri Lanka
Department of Mathematics
B.Sc/ B.Ed Degree Programme
Final Examination - 2019/ 2020
Applied Mathematics– Level 05
ADU5300/ APU3141 – Linear Programming

DURATION: - TWO HOURS

Date: 13 – 01 – 2020	**Time: 01.30 p.m. – 03.30 p.m.**

ANSWER FOUR QUESTIONS ONLY.

01. Products 1 and 2 are produced by use of three machines: A, B and C. Each unit of product 1 requires 1 hour on machine A, 2 hours on machine B and no hours on machine C. Each unit of product 2 requires 1 hour on each machine. The time available on these three machines A, B and C is limited to 400, 600 and 300 hours per month respectively. Each unit of product 1 can be sold to yield a profit of Rs 50 and each unit of product 2 can be sold to yield a profit of Rs 80. Let X_1 and X_2 be the number of units per month to be produced by product 1 and 2 respectively.

(a) By stating the *constraints* clearly, model this as a linear programming problem to *maximize* the total profit (Z) under the given conditions.

(b) A table of *simplex* method for the above stated problem is given below:

Basic	X_1	X_2	S_1	S_2	S_3	Solution
X_1	1	0	1	0	−1	100
S_2	0	0	−2	1	1	100
X_2	0	1	0	0	1	300
Z	0	0	50	0	30	29000

(Here, S_1, S_2, and S_3 represent the slack variables of the three constraints)

Hence, (i) determine the *feasibility* and the *optimality* of this solution.

(ii) how many units of each product are to be produced per month to *maximize* the profit and what is the maximum profit according to this solution?

(c) Verify the solution that you obtained in part (b) of the formulated problem in part (a) by means of *graphical* method.

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02. Define the *degeneracy* in linear programming.

(a) Solve the following linear programming problem using the *simplex* method:

$$\begin{aligned} \textit{Maximize} \quad & z = 3x_1 + 4x_2 \\ \textit{Subject to} \quad & 2x_1 + x_2 \leq 8, \\ & -x_1 + 2x_2 \leq 6, \\ & x_1 + x_2 \leq 6, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(b) In part (a), identify the *constraint* that causes *degeneracy* and solve the problem *graphically* after dropping this *constraint* from the original problem. Hence, show that the *degeneracy* disappears and the same *optimal* solution is obtained.

03. (a) Solve the following linear programming problem (LPP) using the *big-* M method:

$$\begin{aligned} \textit{Minimize} \quad & z = 2x_1 + x_2 \\ \textit{Subject to} \quad & x_1 + x_2 = 2, \\ & x_1 \geq 1, \\ & x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(b) Write down the model for *phase 1* of the *two-phase* method of the above LPP.

(c) Construct the *initial tableau* of the model for ***phase 2*** of *two-phase* method of the above LPP.

It is given that the *basic feasible* solution of the model for *phase 1* in part (b) as follows:

Basic	x_1	x_2	*Surplus Variable for constraint 2*	*Artificial Variable for constraint 1*	*Artificial Variable for constraint 2*	*Solution*
x_2	0	1	1	1	−1	1
x_1	1	0	−1	0	1	1
z	0	0	0	1	1	0

(i) Hence, obtain the *optimal* solution of *phase 2* of the *two-phase* simplex method.

(ii) Verify that the *optimal* solutions obtained using *big-M* and *two-phase* methods are same.

04. Describe a scenario, in the initial table of *simplex* method, that is both *primal* and *dual feasible*.

(a) Using the dual *simplex method*, find the optimal solution of the following *primal* problem:

$$\textit{Minimize } z = 2x_1 + 3x_2 + 4x_3$$

$$\textit{Subject to} \quad x_1 + 2x_2 + x_3 \geq 4,$$

$$2x_1 - x_2 + 3x_3 \geq 3,$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$$

(b) Write down the *dual* of the above *primal problem* and solve it *graphically*.

Hence, verify that the *optimal* solutions of the objective functions of the *primal* and *dual* problems are equal.

05. A coach of a cricket team has to allocate six top order batting positions (*I, II, III, IV, V* and *VI*) to six batsmen (*A, B, C, D, E* and *F*) in a play-off of a tournament series. The runs scored by each batsman during the five initial rounds of the tournament series at these six positions are given in the following table:

	I	*II*	*III*	*IV*	*V*	*VI*
A	32	35	25	60	50	40
B	50	62	35	40	47	38
C	35	40	32	20	50	45
D	25	30	22	20	30	27
E	50	65	70	60	75	55
F	40	60	45	50	60	50

(a) By clearly defining the decision variables and stating the constraints, model this as an *assignment* problem to *maximize* the total score at the play-off match.

(b) Find the best position for each batsman in order to get 322 as the total score at the play-off match to qualify for the final match of the series.

06. A manufacturing concern has decided to produce three new products A, B and C at four similar factories I, II, III and IV. The unit manufacturing cost (in Rs) of each product at the four factories is displayed in the following table:

	I	II	III	IV
A	3	6	8	4
B	6	1	2	5
C	7	8	3	9

Sales forecast indicates that 20, 28 and 17 units of product A, B, and C respectively should be produced per day. The factories can produce at the most 15, 19, 13 and 18 units respectively of the products per day.

(a) Formulate the above problem as a transportation problem in order to *minimize* the total manufacturing cost.

(b) To apply transportation algorithm, find the *initial basic feasible solution* (IBFS) using each of the following methods separately:

(i) *North- West Corner,*

(ii) *Minimum- Cost,*

(iii) *Vogel's Approximation.*

(c) By selecting the best IBFS in part (a), apply the *transportation algorithm* to show that the *optimal* manufacturing cost is Rs 200.