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THE OPEN UNIVERSITY OF SRI LANKA

Department of Electrical and Computer Engineering
Bachelor of Technology

Final Examination 2014 /2015

ECX 6234 – Digital Signal Processing

Closed Book Test

Date: 05 September 2015 **Time 09:30 -12:30**

Answer five questions selecting at least one question from section B.

Section A

Question 1

(a) The continuous-time signal $x(t) = \sin(20\pi t) + \text{Cos}(40\pi t)$ is sampled with a sampling period T to obtain the discrete-time signal

$$x(n) = sin\left(\frac{\pi n}{5}\right) + cos\left(\frac{2\pi n}{5}\right)$$

Determine a choice of T.

[4 marks]

(b) A system is defined by the input output relation as,

$$y(n) = \cos[x(n)]$$

Find the system is,

(i) Linear
(ii) Time – invariant
(iii) Causal

[6 marks]

(c) Explain what is meant by saying that a linear time invariant system is "BIBO stable" Prove that if a linear time invariant system is BIBO stable, then its impulse response, $h[n]$, satisfies $\sum_{n=-\infty}^{\infty}|h[n]| < \infty$

Determine whether the system given in Q1(b) is BIBO stable.

[4 marks]

(d) You are given the two signals $x_1[n]$ and $x_2[n]$ shown below. The signal values are zero outside of the range of n as shown in figure 1.

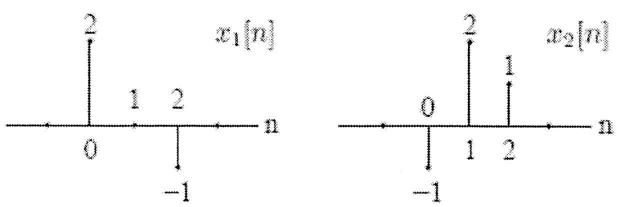

Figure 1

(i) Use graphical methods to find the linear convolution of $x_1[n]$ and $x_2[n]$.
(ii) Using z-transform convolution property verify your results obtained in Q1(d)(i)

[6 marks]

Question 2

(a)Explain what is meant by the Region of Convergence of the z-transform.

[3 marks]

(b) List similarities and differences between the z-transform and the Laplace transform.

[3 marks]

(c) Consider a casual LTI system with the system function.

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}}$$

Where a is real.

(i) Write a difference equation that relates the input and output of this system.
(ii) Find the rage of values of a where system is stable.
(iii) For $a = 1/2$ plot the pole-zero diagram and show ROC.
(iv) Find the impulse response $h[n]$ for the system when $a = 1/2$
(v) Determine and plot the magnitude response of this system for $a = 1/2$. Hence deduce the filter type.

[10 marks]

(d) Consider a filter with the following system function:

$$H(z) = \frac{z^2 + \frac{1}{4}z}{(1 - \frac{1}{2})^2}, \qquad |z| > \frac{1}{2}$$

Using z transformer properties find the impulse response of the system.
Hint:

$$Z\{na^n u[n]\} = \frac{az^{-1}}{(1 - az^{-1})^2} \quad |z| > |a|$$

$$Z\{x[n - n_0]\} = z^{-n_0}X(z)$$

[4 marks]

Question 3

Two discrete-time sequences are given below.

$$x[n] = \delta[n-2] + 3\delta[n] + 2\delta[n-1] + 5\delta[n-3]$$

$$y[n] = \delta[n] - 2\delta[n-3]$$

(a) Write an expression for the 4-point DFT $X[k]$ of $x[n]$, and find the value $X[1]$.

[4 marks]

(b) Find the 4-point inverse DFT of $X[k]e^{-j\frac{2\pi}{4}}$, where $X[k]$ is the 4 point DFT of $x[n]$

[4 marks]

(c) Find the 4-point circular convolution of $x[n]$ with $y[n]$

[4 marks]

(d) Redo Q3(c) using the DFT.

[4 marks]

(e) Explain how FFT can be used to calculate linear convolution of $x[n]$ and $y[n]$.

[4 marks]

Question 4

(a) Produce signal-flow-graphs for each of the following difference equations:

(i) $y[n] = x[n] - x[n-2]$
(ii) $y[n] = x[n] - x[n-1] - y[n-1]$

For each of the difference equations above, determine its impulse-response and deduce whether the discrete-time system it represents is stable and/or causal.

[6 marks]

(b) Consider the block diagram realization given in figure 2.

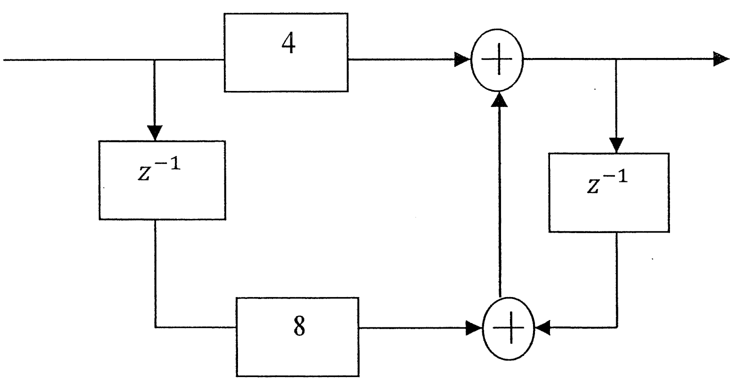

Figure 2

Express the system in the state-space form.
i.e. Find the matrices A,B,C and D such that,

$$s[n+1] = [A]s[n] + [B]x[n]$$

$$y[n] = [C]s[n] + [D]x[n]$$

$s[n]$ is the state vector and $y[n]$ is the Output vector

[8marks]

(c) State-space equation for a certain system is given by

$$s[n+1] = 0.6s[n] + x[n]$$

$$y[n] = 3s[n]$$

(i) Find the transfer function H(z)

(ii) Find the impulse response h(z)

[6 marks]

Section B

Question 5

(a) Briefly state the advantages and disadvantages of infinite impulse-response digital filters (IIR) compared with the finite impulse response (FIR) filter type.

[4 marks]

(b) Design a linear low pass fourth order FIR filter using window techniques to meet the following requirements.

Cut off frequency = 2 kHz
Window type = Rectangular
Sampling Frequency = 20kHz

[8 marks]

(c) Find the transfer function and impulse response of the filter in Q5 (b)

[4 marks]

(d) Draw the signal flow graph for above filter.

[4 marks]

Question 6

It is required to eliminate an unwanted sinusoidal component of a digitalized signal without affecting the magnitudes of other frequency components. Following details has given.

Sampling frequency = 3 kHz
Frequency of the unwanted sinusoid = 250 Hz
3dB band-width of the notch should be approximately 38.2 Hz

(a) Design a notch filter by pole and zero placement method.

[10marks]

(b) Find the transfer function H(z). Hence or otherwise deduce difference equation for the filter system

[4marks]

(c) Sketch the gain response indicating important parameters.

[6marks]

Question 7

(a) What is meant by
(i) Upsampling
(ii) Downsampling
Of a digital signal?
Give one example of practical application for each cases.

[4marks]

(b) Discrete signal is given by
x(n) = {3, 1, 4, 1, 5, 9, 2, 6, 5 . . .}, x(0) = 3

Write down the up sampled (by 2) signal $x_u(n)$ and downsampled (by 2) signal $x_d(n)$.

[4marks]

(c) Consider the multirate signal processing operations of decimation, denoted ↓M, and expansion, denoted ↑L, where M and L are the decimation and expansion factors respectively. When the real discrete-time signal $x(n)$ with z transform $X(z)$, is decimated by a factor M, it is denoted $x_D(n)$ and when expanded by a factors L, it is denoted $x_E(n)$. Show that the z-transform of $x_D(n)$ is given by

$$X_D(Z) = \frac{1}{M}\sum_{k=0}^{M-1} X(z^{\frac{1}{M}}W_M^k)$$

[6marks]

(d) Musician has snippet of music at note E. It is required to alter note E to note A. Note A has frequency 2/3 that of note E.
Design a multirate system to do this task.

[6marks]

Question 8

The discrete-time model for a linear stochastic system can be represented by following matrix equations:

$$x_k = \Phi_{k-1}x_{k-1} + G_{k-1}w_{k-1}$$

$$z_k = H_k x_k + v_k$$

Where $\{w_k\}$ and $\{x_k\}$ are zero-mean uncorrelated Gausian random processes.

(a) Draw a block diagram of the system. On your block diagram show the system model and the measurement model.

[4marks]

(b) An estimator $\hat{x}_k$ is derived from the system and fed to a Kalman filter.

(i) Describe the function of a Kalman filter.

[5marks]

(ii) How does filter calculate the best value for $\hat{x}_k$? Explain the principle behind this process.

[5marks]

(iii) If the gain matrix of the Kalman filter is $\overline{K}_k$ draw a block diagram of the Kalman filter.

[6marks]

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