The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Title and - Code	: Newtonian Mechanics I – ADU4301/APU2142
Academic Year	: 2019/20
Date	: 29.10.2020
Time	: 1.30 p.m. To 3.30. p.m.
Duration	: Two Hours.

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (4) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1. At time $t = 0$, an insect of mass m jumps from a point O on the ground with velocity $\underline{v}_0$, while a wind blows with constant velocity $\underline{u}$. The gravitational acceleration is $\underline{g}$ and the air exerts a drag force on the insect equal to mk times the velocity of wind relative to the insect.

   (a) Show that the velocity of the insect is given by $\underline{v} = \left(\underline{u} + \frac{\underline{g}}{k}\right) + \left(\underline{v}_0 - \underline{u} - \frac{\underline{g}}{k}\right)e^{-kt}$

   (b) Show that the path of the insect is given by $\underline{r} = \left(\underline{u} + \frac{\underline{g}}{k}\right)t + \frac{\left(1 - e^{-kt}\right)}{k}\left(\underline{v}_0 - \underline{u} - \frac{\underline{g}}{k}\right)$.

   (c) In the case where the insect jumps up vertically in a horizontal wind, show that the time T that elapses before it returns to the ground (which is also horizontal) satisfies

   $1 - e^{-kT} = \frac{kT}{1 + \lambda}$, where $\lambda = \frac{kv_0}{g}$. Find an expression for horizontal range R in terms of λ, u and T. (Here $v_0 = |\underline{v}_0|$, $g = |\underline{g}|$ and $u = |\underline{u}|$.

2. (a) With the usual notation, show that in intrinsic coordinates, the velocity and acceleration $\underline{a}$ of a particle moving in a plane curve are given by $\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\,\underline{n}$ respectively.

   (b) A smooth wire in the form of an arch of the cycloid, with intrinsic equation $s = 4a\sin\psi$, is fixed in a vertical plane with its vertex downwards. The tangent at the vertex is horizontal. A small bead, of mass m, is threaded onto the wire and is subject to an air resistance of magnitude $\frac{mv^2}{8a}$ when its speed is v, this resistance being always directly opposite to the direction of motion.

   Given that the bead is projected from the vertex with speed $\sqrt{8ag}$, show that the bead comes to instantaneous rest at a cusp where $s = 4a$.

3. (a) With the usual notation, show that in plane polar coordinates, the velocity $\underline{v}$ and acceleration $\underline{a}$ of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

   $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$ respectively.

(b) The only force acting on a body, which is of mass m and is at a distance r from the centre of the Earth, is directed towards the centre of the Earth and is of magnitude $\frac{\mu m}{r^2}$, where μ is a constant. Show that the speed of a satellite of mass m moving in a circular orbit of radius a about the centre of the Earth is $\sqrt{\frac{\mu}{a}}$

A second satellite, of mass $3m$ subject to the same force, is moving in the same circular orbit as the first but in the opposite direction and the two satellites collide and coalesce to form a single composite body. Show that the subsequent motion of the composite body is governed by the two equations: $r^2\dot{\theta} = \sqrt{\frac{a\mu}{4}}$, $\ddot{r} = \frac{\mu(a-4r)}{4r^3}$ where (r, θ) are the polar coordinates of the body with the centre of the Earth as pole. Find the values of r when $\dot{r}^2 = 0$.

4. (a) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} - \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being added at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(b) A spherical raindrop falls under gravity through a stationary cloud. Initially the drop is at rest and its radius is a. As it falls, water from the cloud condenses on the drop in such a way that the radius of the drop increases at a constant rate k. At time t, the speed of the drop is v.

($\underline{\alpha}$) Show that $(a+kt)\frac{dv}{dt} + 3kv = g(a+kt)$.

(β) Hence show that, when the drop has doubled its radius, its speed is $\frac{15ga}{32k}$.

5. A uniform rod AB of length $4a$ and mass m is free to rotate in a vertical plane about a smooth, fixed, horizontal axis through the point C of the rod where $AC = a$. The rod is released from rest with AB horizontal. When the rod first becomes vertical, end B strikes a stationary particle of mass m which then adheres to the rod. Find, in terms of g and a, the angular speed of the rod after the impact and show that the angle α between the rod and the downward vertical when the rod comes to instantaneous rest is given by $136\cos\alpha = 129$.

6. A uniform rod of mass m and length $2a$ is held at an angle β to the vertical with its lower end on a smooth horizontal plane and is then released.

   (a) Show that the angular speed ω of the rod just before it strikes the plane is given by $2a\omega^2 3g\cos\beta$.

   (b) Find the normal contact force just before it strikes the plane.

   (c) Show that the angular acceleration at that instant is independent of β.