00262

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Code and Title	: ADU4303 Applied Linear Algebra & Differential Equations /APU2144
Academic Year	: 2019/2020
Date	: 06.11.2020
Time	: 09.30a.m.-11.30a.m.
Duration	: 2 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 06 questions in 04 pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
5. Relevant log tables are provided where necessary.
6. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
7. Use blue or black ink to answer the questions.
8. Circle the number of the questions you answered in the front cover of your answer script.
9. Clearly state your index number in your answer script.

1. (i) Define each of the following:
   (a) Symmetric matrix,
   (b) Skew-symmetric matrix,
   (c) Orthogonal matrix,
   (d) Inverse of a matrix.

   (ii) Find the inverse of the matrix B where

$$B=\begin{pmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$

   (iii) Find values of α , for which the following system of equations is consistent and has non-trivial solutions:

$$\begin{aligned} (\alpha-1)x + (3\alpha+1)y + \quad 2\alpha z &= 0 \\ (\alpha-1)x + (4\alpha-2)y + (\alpha+3)z &= 0 \\ 2x + (3\alpha+1)y + 3(\alpha-1)z &= 0 \end{aligned}$$

   (iv) Solve the system of equations,

$$\begin{aligned} &6x_3 + 2x_4 - 4x_5 - 8x_6 = 8 \\ &3x_3 + x_4 - 2x_5 - 4x_6 = 4 \\ &2x_1 - 3x_2 + x_3 + 4x_4 - 7x_5 + x_6 = 2 \\ &6x_1 - 9x_2 + 11x_4 - 19x_5 + 3x_6 = 1 \end{aligned}$$

2. (i) Let $A=\begin{pmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{pmatrix}$.

   (a) Show that $A^3 + (a^2+b^2+c^2)A = (0)$ where (0) is the zero matrix.
   (b) Does this imply that $A^2 + (a^2+b^2+c^2)I = (0)$? Justify your answer.
   (c) Find the adjoint of the matrix A.

   (ii) Transform the following quadratic form to a canonical form by an orthogonal transformation and state the corresponding model matrix.
   $5x_1^2 + 26x_2^2 + 10x_3^2 + 4x_2x_3 + 6x_1x_2 + 14x_1x_3$.

3. Find the general solution of the following systems of simultaneous differential equations:

   (i)
$$\begin{aligned} \dot{x}_1 &= x_1 - x_2 - x_3 \\ \dot{x}_2 &= x_1 + 3x_2 + x_3 \\ \dot{x}_3 &= -3x_1 + x_2 - x_3 \end{aligned}$$

(ii) $\dfrac{dx_1}{dt} = x_1 + 2x_2 + e^t$

$\dfrac{dx_2}{dt} = 2t - x_1 + 4x_2$

(iii) $\ddot{x} = -\dfrac{25}{7}x - \dfrac{4}{7}y$

$\ddot{y} = \dfrac{11}{7}x - \dfrac{10}{7}y$

4. (i) Find a sinusoidal particular solution for the following system of partial differential equations.

$$\ddot{x}_1 + 4x_1 + 2x_2 = 6\cos 2t$$
$$\ddot{x}_2 + x_1 + 9x_2 = 2\sin 2t.$$

(ii) Find the general solution of the following differential equation:

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x^3$$

(iii) (a) Find the general solution of the following pair of partial differential equations:

$$\frac{\partial u}{\partial y} = 5y^4x - p\cos py$$

$$\frac{\partial u}{\partial x} = y^5 + xe^x$$

(b) Find the general solution of the following partial differential equation by using the integrating factor method. (u is a function of the two variables x and y.)

$$\frac{\partial u}{\partial x} - u\tan x = \cos x$$

5. (i) Let u be a function that satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{y}\frac{\partial u}{\partial y} - 4y^2\frac{\partial^2 u}{\partial x^2} = 0, \text{ where } y \neq 0$$

Find u if

(a) u is a function of x only,
(b) u is a function of y only.

(ii) Find the equations of the characteristic curves for the partial differential equations

$$-2xy\frac{\partial u}{\partial x}+4x\frac{\partial u}{\partial y}+yu=4xy; \quad x>0,\ y>0.$$

Hence define the new variable that could be used to simplify the partial differential Equation and solve the equation in terms of x and y.

6. (i) State the conditions for the following differential equation

$$A(x,y)\frac{\partial^2 u}{\partial x^2}+B(x,y)\frac{\partial^2 u}{\partial x\partial y}+C(x,y)\frac{\partial^2 u}{\partial y^2}=F(x,y)$$

to be classified as either hyperbolic, parabolic or elliptic.

(ii) Consider the partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2}-y^2\frac{\partial^2 u}{\partial y^2}=y\frac{\partial u}{\partial y}-x\frac{\partial u}{\partial x}$$

(a) Show that the characteristics are defined by the pair of ordinary differential equations

$$\frac{dy}{dx}=\pm\frac{y}{x},$$

and hence find the characteristics.

(b) Hence show that the above equation can be rewritten in the form

$$\frac{\partial^2 u}{\partial\varsigma\partial\eta}=0,$$

where ς and η are the characteristic variables, and thus find the general solution of above equation.

(iii) Show on a diagram those regions of the (x, y) plane in which the equation

$4y^2\frac{\partial^2 u}{\partial x^2}+4xy\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 u}{\partial y^2}=0$ is classified as being hyperbolic, parabolic and elliptic.