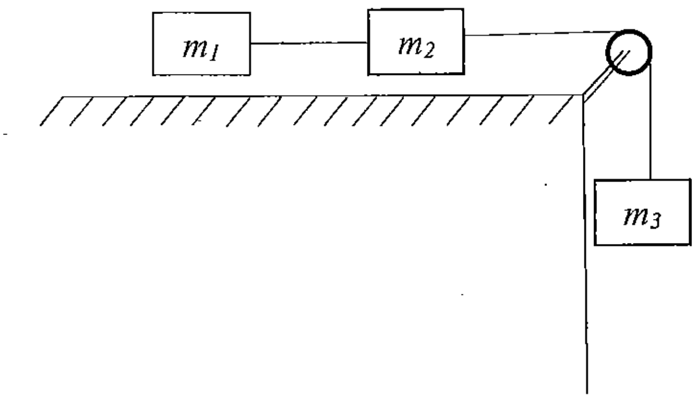
The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	: Final Examination
Course Title and - Code	**: Newtonian Mechanics II – ADU5303/APU3145**
Academic Year	**: 2019/20**
Date	**: 21.12.2020**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in **(3)** pages.
3. Answer any **(4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1. Three blocks of masses m_1, m_2 and m_3 are connected as shown in the figure below. The coefficient of friction between the sliding surfaces of the blocks m_1 and m_2 and the plane are μ_1 and μ_2 respectively. Assuming that mass m_3 is moving vertically downwards, find the acceleration of masses and the tension in the strings using D'Alembert's principle.



2. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, 2 \ldots, n.$

(b) A bead slides without friction on a smooth wire in the shape of a cycloid with equation $x = a(\theta - \sin\theta), \quad y = a(1 + \cos\theta)$ where $0 \le \theta \le 2\pi$, y-axis being vertically upwards. Obtain the Lagrangian function and using that, show that $\ddot{x} = 0$ and $\ddot{y} = -g$.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

(b) A projectile located at a point of latitude λ is projected with speed v_0 in a southward direction at an angle α to the horizontal. Find the position of the projectile after time t. Prove that after time t, the projectile will be deflected towards the east of the original vertical plane of motion by the amount $\frac{1}{3}\omega g \cos\lambda t^3 - \omega v_0 \sin(\alpha + \lambda) t^2$.

4. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) A body moves about a point O under no forces. The principle moment of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n,\ \omega_2 = 0,\ \omega_3 = n$ about the corresponding principal axes. Show that at any time t,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tanh\left(\frac{nt}{\sqrt{5}}\right)$$

5. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i,\quad \frac{\partial H}{\partial q_i} = -\dot{p}_i$.

(b) The Hamiltonian of a dynamical system is given by $H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$ where a, b are constants. Obtain Hamilton's equations of motion and hence find p_1, q_1, p_2 and q_2 at time t.

6. (*a*) Define Canonical Coordinates and Canonical Transformations.

(b) Show that there exists a function F which generates the Canonical Transformation.

(c) The transformation equations between two sets of coordinates are $Q = \log\left(1 + q^{1/2}\cos p\right)$ and $P = 2\left(1 + q^{1/2}\cos p\right) q^{1/2}\sin p = 2\,q^{1/2}\sin p + q\sin 2p$.

Show that

(i) these transformations are canonical if q and p are canonical and

(ii) the function which generates the above transformation is $F_3 = -\left(e^{Q} - 1\right)^2 \tan p$.