The Open University of Sri Lanka
B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME
Final Examination - 2019/2020
Level 05 - Applied Mathematics
ADU5305/APU3147/ADE5305– Statistical Inference

**Duration: - Two Hours.**

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**DATE: - 16-12-2020** **Time: - 9.30 a.m. to 11.30 a.m.**

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**Non programmable calculators are permitted. Statistical tables are provided.**
**Answer only four questions.**

1.

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a distribution with density given by $f(x;\theta)$. Let $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3, \hat{\theta}_4$, are functions of $X_1, X_2, X_3, \ldots X_n$. Suppose $\hat{\theta}_1, \hat{\theta}_2$ are asymptotically unbiased and precise estimators for parameter $\theta$, $\hat{\theta}_3$ consistent estimator for parameter $\theta$, and $\hat{\theta}_4$, is unbiased estimator for parameter $\theta$. State whether the following statements are true or false. Justify your answer.

(i) $Bias(\frac{\hat{\theta_4}}{2} + 2) = 2$

(ii) $Var(\hat{\theta}_3) > 0$ for large samples

(iii) $\frac{\hat{\theta_1}+\hat{\theta_4}}{2}$ is accurate estimator for large samples

(iv) $\hat{\theta}_2$ is a consistent estimator for $\theta$.

(v) $\hat{\theta}_3$ is an accurate and precise estimator for small samples.

2.

Suppose weight $(X)$ of a certain product produced by ABC Company, follows a normal distribution. However, the mean and variance of $X$ are unknown.
Let $X_1, X_2, X_3, \ldots X_n$ be the weights of randomly selected $n$ products in grams.

(i) Find the Moment estimators for the mean and the variance of weight of a randomly selected product.

(ii) Weights of 10 randomly selected products in grams are given bellow. Using part (i) estimate the mean and the variance weight of a randomly selected product.

| | | | | |
|---|---|---|---|---|
| 54.6 | 49.7 | 46.3 | 49.1 | 51.2 |
| 52.0 | 49 | 51.5 | 47.6 | 51.5 |

(iii) Using a suitable statistical test comment on the claim that "mean of a randomly selected product is 50g". Use 5% level of significance.

3.

Assignment marks and final examination marks of a subject for randomly selected 15 students are given below. From the past experience it is reasonable to assume that Assignment mark and Final Mark follow normal distributions.

| Student Name | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| Assignment mark | 60 | 47 | 60 | 56 | 47 | 27 | 45 | 61 | 68 | 62 |
| Final Mark | 67 | 54 | 53 | 49 | 47 | 35 | 30 | 77 | 57 | 54 |

(i) Construct a 95% confidence interval for mean of the Assignment marks.

(ii) Construct a 95% confidence interval for variance of the Assignment marks.

(iii) Using suitable statistical test, examine the validity of the claim that "Expected assignment mark is greater than the expected final examination mark for a randomly selected student". Use 5% level of significance.

4.

In a process of a production, the production manager is interested in the proportion $\theta$ of defective items produced. Suppose a random sample of 100 items (drawn with replacement) were tested. Suppose that 5 items were defective.

(i) Construct a 95% confidence interval for $\theta$. Interpret the results.

(ii) Comment on the claim that ( "95% of the produced items are not defective")

(iii) Using a suitable statistical test comment on the claim that "proportion of defective items of the production process $\theta$ is greater than 0.11"

5.

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{\theta} \quad ; \; 0 \leq x \leq \theta ; \quad \theta > 0$$

(i) Prove that the mean and variance of the above distribution is $\frac{\theta}{2}$ and $\frac{\theta^2}{12}$.

(ii) Derive the maximum likelihood estimator for $\theta$ .

(iii) Derive maximum likelihood estimator for the mean and variance of the above distribution.

(iv) A random sample drawn from the above distribution is given below.

| | | | | |
|---|---|---|---|---|
| 0.49 | 1.38 | 1.95 | 1.28 | 1.80 |
| 0.21 | 0.05 | 0.48 | 0.49 | 1.40 |
| 0.59 | 0.23 | 0.26 | 1.65 | 1.43 |
| 1.56 | 0.16 | 0.82 | 0.37 | 0.80 |

Estimate the mean and variance of the above distribution using maximum likelihood estimators derived in part (iii)

6.

Briefly explain the following terms.

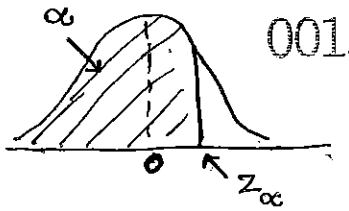
(i) Accuracy and Precision

(ii) Sampling distribution

(iii) Confidence Interval

(iv) Null Hypothesis and Alternative Hypothesis

(v) Statistical Test

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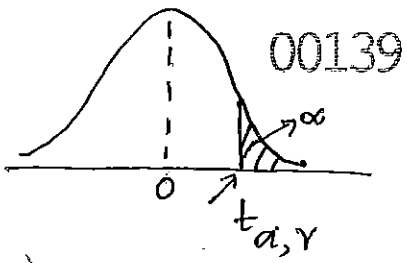
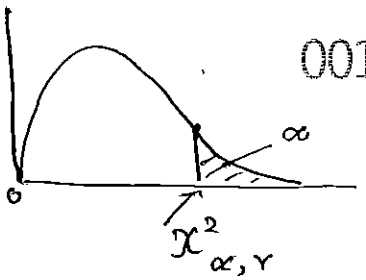
**STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.**

| Z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .50000 | .50399 | .50798 | .51197 | .51595 | .51994 | .52392 | .52790 | .53188 | .53586 |
| 0.1 | .53983 | .54380 | .54776 | .55172 | .55567 | .55962 | .56356 | .56749 | .57142 | .57535 |
| 0.2 | .57926 | .58317 | .58706 | .59095 | .59483 | .59871 | .60257 | .60642 | .61026 | .61409 |
| 0.3 | .61791 | .62172 | .62552 | .62930 | .63307 | .63683 | .64058 | .64431 | .64803 | .65173 |
| 0.4 | .65542 | .65910 | .66276 | .66640 | .67003 | .67364 | .67724 | .68082 | .68439 | .68793 |
| 0.5 | .69146 | .69497 | .69847 | .70194 | .70540 | .70884 | .71226 | .71566 | .71904 | .72240 |
| 0.6 | .72575 | .72907 | .73237 | .73565 | .73891 | .74215 | .74537 | .74857 | .75175 | .75490 |
| 0.7 | .75804 | .76115 | .76424 | .76730 | .77035 | .77337 | .77637 | .77935 | .78230 | .78524 |
| 0.8 | .78814 | .79103 | .79389 | .79673 | .79955 | .80234 | .80511 | .80785 | .81057 | .81327 |
| 0.9 | .81594 | .81859 | .82121 | .82381 | .82639 | .82894 | .83147 | .83398 | .83646 | .83891 |
| 1.0 | .84134 | .84375 | .84614 | .84849 | .85083 | .85314 | .85543 | .85769 | .85993 | .86214 |
| 1.1 | .86433 | .86650 | .86864 | .87076 | .87286 | .87493 | .87698 | .87900 | .88100 | .88298 |
| 1.2 | .88493 | .88686 | .88877 | .89065 | .89251 | .89435 | .89617 | .89796 | .89973 | .90147 |
| 1.3 | .90320 | .90490 | .90658 | .90824 | .90988 | .91149 | .91309 | .91466 | .91621 | .91774 |
| 1.4 | .91924 | .92073 | .92220 | .92364 | .92507 | .92647 | .92785 | .92922 | .93056 | .93189 |
| 1.5 | .93319 | .93448 | .93574 | .93699 | .93822 | .93943 | .94062 | .94179 | .94295 | .94408 |
| 1.6 | .94520 | .94630 | .94738 | .94845 | .94950 | .95053 | .95154 | .95254 | .95352 | .95449 |
| 1.7 | .95543 | .95637 | .95728 | .95818 | .95907 | .95994 | .96080 | .96164 | .96246 | .96327 |
| 1.8 | .96407 | .96485 | .96562 | .96638 | .96712 | .96784 | .96856 | .96926 | .96995 | .97062 |
| 1.9 | .97128 | .97193 | .97257 | .97320 | .97381 | .97441 | .97500 | .97558 | .97615 | .97670 |
| 2.0 | .97725 | .97778 | .97831 | .97882 | .97932 | .97982 | .98030 | .98077 | .98124 | .98169 |
| 2.1 | .98214 | .98257 | .98300 | .98341 | .98382 | .98422 | .98461 | .98500 | .98537 | .98574 |
| 2.2 | .98610 | .98645 | .98679 | .98713 | .98745 | .98778 | .98809 | .98840 | .98870 | .98899 |
| 2.3 | .98928 | .98956 | .98983 | .99010 | .99036 | .99061 | .99086 | .99111 | .99134 | .99158 |
| 2.4 | .99180 | .99202 | .99224 | .99245 | .99266 | .99286 | .99305 | .99324 | .99343 | .99361 |
| 2.5 | .99379 | .99396 | .99413 | .99430 | .99446 | .99461 | .99477 | .99492 | .99506 | .99520 |
| 2.6 | .99534 | .99547 | .99560 | .99573 | .99585 | .99598 | .99609 | .99621 | .99632 | .99643 |
| 2.7 | .99653 | .99664 | .99674 | .99683 | .99693 | .99702 | .99711 | .99720 | .99728 | .99736 |
| 2.8 | .99744 | .99752 | .99760 | .99767 | .99774 | .99781 | .99788 | .99795 | .99801 | .99807 |
| 2.9 | .99813 | .99819 | .99825 | .99831 | .99836 | .99841 | .99846 | .99851 | .99856 | .99861 |
| 3.0 | .99865 | .99869 | .99874 | .99878 | .99882 | .99886 | .99889 | .99893 | .99896 | .99900 |
| 3.1 | .99903 | .99906 | .99910 | .99913 | .99916 | .99918 | .99921 | .99924 | .99926 | .99929 |
| 3.2 | .99931 | .99934 | .99936 | .99938 | .99940 | .99942 | .99944 | .99946 | .99948 | .99950 |
| 3.3 | .99952 | .99953 | .99955 | .99957 | .99958 | .99960 | .99961 | .99962 | .99964 | .99965 |
| 3.4 | .99966 | .99968 | .99969 | .99970 | .99971 | .99972 | .99973 | .99974 | .99975 | .99976 |
| 3.5 | .99977 | .99978 | .99978 | .99979 | .99980 | .99981 | .99981 | .99982 | .99983 | .99983 |
| 3.6 | .99984 | .99985 | .99985 | .99986 | .99986 | .99987 | .99987 | .99988 | .99988 | .99989 |
| 3.7 | .99989 | .99990 | .99990 | .99990 | .99991 | .99991 | .99992 | .99992 | .99992 | .99992 |
| 3.8 | .99993 | .99993 | .99993 | .99994 | .99994 | .99994 | .99994 | .99995 | .99995 | .99995 |
| 3.9 | .99995 | .99995 | .99996 | .99996 | .99996 | .99996 | .99996 | .99996 | .99997 | .99997 |




# Table of $t_{\alpha,\nu}$ quantiles (t-table)

| df | $\alpha$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $\nu$ | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.001 | 0.0005 |
| 1 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| 2 | 1.886 | 2.92 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.61 |
| 5 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | 1.44 | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | 1.397 | 1.86 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | 1.383 | 1.833 | 2.262 | 2.821 | 3.25 | 4.297 | 4.781 |
| 10 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.93 | 4.318 |
| 13 | 1.35 | 1.771 | 2.16 | 2.65 | 3.012 | 3.852 | 4.221 |
| 14 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.14 |
| 15 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |




# Table of $\chi^2_{\alpha,\nu}$ quantiles ($\chi^2$ table)

| df | $\alpha$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\nu$ | 0.99 | 0.975 | 0.95 | 0.90 | 0.1 | 0.05 | 0.025 | 0.01 |
| 1 | 0 | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 |
| 2 | 0.02 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.21 |
| 3 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 |
| 4 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 |
| 5 | 0.554 | 0.831 | 1.145 | 1.61 | 9.236 | 11.07 | 12.833 | 15.086 |
| 6 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 |
| 7 | 1.239 | 1.69 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 |
| 8 | 1.646 | 2.18 | 2.733 | 3.49 | 13.362 | 15.507 | 17.535 | 20.09 |
| 9 | 2.088 | 2.7 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 |
| 10 | 2.558 | 3.247 | 3.94 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 |
| 11 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.92 | 24.725 |
| 12 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 |
| 13 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 |
| 14 | 4.66 | 5.629 | 6.571 | 7.79 | 21.064 | 23.685 | 26.119 | 29.141 |
| 15 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 |
| 16 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32 |
| 17 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 |
| 18 | 7.015 | 8.231 | 9.39 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 |