00085

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: ADU5306/APU3150- Fluid Mechanics**
Academic Year	**: 2019/20**
Date	**: 19.12.2020**
Time	**: 09.30 a.m – 11.30 a.m**
Duration	**: 2 hours**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **06** questions in **03** pages.
3. Answer **04** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1.

(a) Briefly describe the each of the followings: **[8 Marks]**

i. Steady and Un-steady flows
ii. Uniform and Non-uniform flows
iii. Compressible and Incompressible flows
iv. Rotational and Irrotational flows.

(b) The fluid flow field is given by $\boldsymbol{q} = x^2y\boldsymbol{i} + y^2z\boldsymbol{j} - (2xyz + z^2y)\boldsymbol{k}$. Show that this is a case of a possible steady, incompressible flow field. **[9 Marks]**

(c) Show whether the fluid flow field given by $\boldsymbol{q} = e^x[(\sin z - \cos y)\boldsymbol{i} + \sin y\boldsymbol{j} + \cos z\,\boldsymbol{k}]$ is irrotational. **[8 Marks]**

2.

(a) Show that $q = (\alpha y, \alpha x - \beta, 0)$ represents the velocity of an incompressible fluid in an irrotational motion, where α, β are constants. Also,

i. find the velocity potential, and
ii. Obtain that streamlines are given by the curves of intersection of $\alpha(x^2 - y^2) - 2\beta x =$ constant and $z =$ constant. **[12 Marks]**

Show that equation of continuity can be reduced as $\nabla^2\emptyset = 0$ for an incompressible fluid in an irrotational motion. Here $\emptyset$ represents the velocity potential. Show that

$$\emptyset = \frac{x}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3}$$

represents a possible motion satisfying the above reduced form.

Then what would be the fluid velocity? **[13 Marks]**

3.

Derive the continuity equation of the form $\frac{D\rho}{Dt} + \rho div(\underline{q})=0$, for any arbitrary control volume of a moving fluid irrespective of its shape and size. **[8 Marks**

(a) Hence deduce the continuity equation, for an incompressible fluid in terms of Cartesian Coordinates. **[5 Marks]**

(b) Consider the fluid flow field which is given by $\underline{q} = x^2y\underline{i} + y^2z\underline{j} - (2xyz + yz^2)\underline{k}$. Prove that this is a case of a possible incompressible flow field. **[6 Marks]**

(c) Given $v = 2y^2$ and $w = 2xyz$, the two velocity components. Determine the third component such that it satisfies the continuity equation. **[6 Marks]**

4.

(a) Given Euler's equation of motion $\underline{F} - \frac{1}{\rho} grad p = \frac{Dq}{Dt}$ for a perfect fluid, show that it can be written in the form $\underline{F} - \frac{1}{\rho} grad p = \frac{\partial q}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{q} \times curl \underline{q}$. **[10 Marks]**

(b) Using the result in Part (a), derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density. **[7 Marks]**

(c) Consider a horizontal nozzle discharging into the atmosphere. The inlet has a bore area of 500 mm^2 and the exit has a bore area of 250 mm^2 . Assuming there is no energy loss calculate the flow rate when the inlet pressure is $400 Pa$. **[8 Marks]**

5.

(a)A fluid is in equilibrium under the external force per unit mass $\underline{F}$ on a flat plate.

i. Show that $\underline{F}.dr = \frac{dp}{\rho}$

ii. If the external forces acting on the fluid is gravitational force only, then show that $-\rho g dz = dp$

iii. Furthermore, if $\rho = \exp(-z)$, then show that $p = p - g(1 - e^{-z})$ where p_0 is the pressure acting on the free space. **[15 Marks]**

(b) Suppose a motion of an incompressible homogeneous fluid under no force is steady. The velocity at any point is given by $byi + ayj - 2azk$, where a is a constant. Find the surface of equal pressure. **[10 Marks]**

6.

A right circular cylinder of radius a stands with its axis vertical and its base attached to a infinite rigid horizontal plane $z = 0$. It is surrounded by an ocean of incompressible non-viscous liquid of infinite extent, bounded below by the plane $z = 0$ and above by its free surface open to the atmosphere at pressure p_0. The cylinder extends above the free surface of the ocean, whose height at a large distance from the cylinder is h. If the velocity components of the liquid at the point (x, y, z) in RCC, are $\left(\frac{wa^2y}{r^2}, -\frac{-wa^2x}{r^2}, 0\right)$ where $r^2 = x^2 + y^2$, show that the motion is irrotational and find the following quantities.

(a) The velocity potential of the motion. **[8 Marks]**

(b) The liquid pressure at a point on the surface of the cylinder at a height z. **[8 Marks]**

(c) The liquid pressure on the plane base $z = 0$, at a distance r from the axis. **[4 Marks]**

(d) The height of the free surface above the plane $z = 0$, as it touches the cylinder. **[5 Marks]**