The Open University of Sri Lanka
Department of Mathematics
B. Sc/ B. Ed Degree Programme
Final Examination - 2019/ 2020
Pure Mathematics– Level 03

PEU3301/PUU1141/PUE3141 – Foundations of Mathematics
Duration: Two Hours

Date: 24 -10 - 2020 **Time: 09.30 a.m.-11.30 a.m.**

ANSWER FOUR (04) QUESTIONS ONLY

Q1

(a) Solve the inequality $|x+2|+|x-3|>5$ where $x \in \mathbb{R}$.

(b) Let R be the relation defined on $\mathbb{Z}$ by xRy if 3 divides $x+2y$.
 (i) Show that R is an equivalence relation on $\mathbb{Z}$.
 (ii) Find the equivalence class of 1.

Q2

(a) Let $m, n \in \mathbb{Z} \setminus \{0\}$. Prove that if there exist $x, y \in \mathbb{Z}$ such that $mx+ny=1$ then m and n are relatively prime.

(b) Let $m, n \in \mathbb{N}$. Prove that $\gcd(m,n)\operatorname{lcm}(m,n)=mn$.
Find $\gcd(546, 422)$ and $\operatorname{lcm}(546, 422)$.

(c) Let $f(x)=2x+3, x \in [2,3]$. Show that f is a bijection from $[2,3]$ to $[7,9]$

Q3

(a) Show that $\sqrt{1+\sqrt{2+\sqrt{3}}}$ is an algebraic number.

(b) Let f and g be the functions given by

$$f(x)=\frac{x-4}{x+2}, x \in \mathbb{R} \setminus \{-2\}, \text{ and } g(x)=\frac{x+3}{x-2}, x \in \mathbb{R} \setminus \{2\}.$$

Find $f \circ g$ and $g \circ f$ with their respective domains.

Q4

(a) Let $A = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$.

(i) Prove that both $\sup A$ and $\inf A$ exist.

(ii) Find $\sup A$ and $\inf A$. Justify your answers.

(b) Solve the inequality $\frac{x-8}{x} \leq 3 - x$ and sketch the solution on the number line.

Q5

(a) Let $< x_n >$ be the sequence of real numbers defined by $x_n = \frac{6}{(2n-1)(2n+1)}$ for $n \in \mathbb{N}$.

(i) Find A and B such that $\frac{6}{(2n-1)(2n+1)} = \frac{A}{(2n-1)} + \frac{B}{(2n+1)}$

(ii) Using the above result, show that the nth partial sum $s_n = \sum_{k=1}^{n} \frac{6}{(2k-1)(2k+1)} = 3 - \frac{3}{2n+1}$, for each $n \in \mathbb{N}$.

(iii) Show that the series $\sum_{n=1}^{\infty} x_n$ converges. Find $\sum_{n=1}^{\infty} x_n$.

(b) Let $f(x) = x^2 + 7, x \in \mathbb{R}$ and let $A = [11, 23)$. Find $f^{-1}(A)$.

Q6

(a) Let A, B and C be sets. Prove by the method of conditional proof that

(i) $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$ and

(ii) $(A \setminus B) \cap (A \setminus C) \subseteq A \setminus (B \cup C)$.

(b) A non-empty set X is said to be countable if there is a sequence $< x_n >$ such that $X = \{x_n : n \in \mathbb{N}\}$. Assuming that the interval $(0, 1)$ is not countable, prove that for $a, b \in \mathbb{R}$ and $a < b$, the interval (a, b) is also not countable.

=========//=========