The Open University of Sri Lanka
Department of Electrical and Computer Engineering
ECX6241 – Field Theory
Final Examination – 2014/2015

Date: 2015-08-26 Time: 0930-1230

Answer **five questions** by selecting **two** from **Section A**, **two** from **Section B** and **one** from **Section C**.

Section A

Select two questions from this section. (15 Marks for each)

Q1.

(a) Transform the vector $\boldsymbol{F} = 10\, r^{-1} \boldsymbol{a}_r$ in Spherical coordinates into Cartesian coordinates. [4]

(b) List three properties of gradient of a scalar. [3]

(c) Let $\boldsymbol{v} = \boldsymbol{\omega} \times \boldsymbol{r}$ where $\boldsymbol{\omega}$ is a constant vector and $\boldsymbol{r} = x\, \boldsymbol{a}_x + y\, \boldsymbol{a}_y + z\, \boldsymbol{a}_z$. Prove that $\boldsymbol{\omega} = \frac{1}{2} curl\, \boldsymbol{v}$. [8]

Q2.

(a) Discuss the physical interpretation of "curl" with aid of an example relevance to the electromagnetic fields. [3]

(b) For a scalar field V, show that $\nabla \times \nabla V = 0$, i.e., the curl of the gradient of any scalar field is zero. [5]

(c) Find the line integral of the vector $\boldsymbol{F} = (x^2 - y^2)\, \boldsymbol{a}_x + 2xy\, \boldsymbol{a}_y$ around a square of side a which has a corner at the origin, one side on the x-axis and the other side on the y-axis. [7]

Q3.

(a) Given that $\boldsymbol{A} = \frac{5r^3}{4}\, \boldsymbol{a}_r\; C/m^2$ in cylindrical coordinates, evaluate both sides of divergence theorem for the volume enclosed by $r = 1\,m, r = 2\,m, z = 0$ and $z = 10\,m$. [8]

(b) Verify the stokes theorem for the vector field $\boldsymbol{F} = \boldsymbol{a}_x + y^2 z\, \boldsymbol{a}_y$, over the flat surface in the yz plane bounded by (0,0,0), (0,1,0), (0,1,1) and (0,0,1). [7]

Section B

Select **two** questions from this section. (20 Marks for each)

Q4.

(a) State the uniqueness theorem. [5]

(b) Given the potential field $V = \frac{50 \sin\theta}{r^2}$ V in free space.

i. Determine whether V satisfies Laplace's equation. [8]

ii. Find the total charge stored inside the spherical shell $1 \leq r \leq 2$. [7]

Q5.

(a) Explain the "fringing effect". [3]

(b) Derive an expression for the capacitance of a spherical capacitor which consists of two concentric spherical shells of radii a and b $(a < b)$. The dielectric medium between the two spheres is air. [10]

(c) Hence show that the same expression can be written as $C = \frac{\varepsilon_0}{d}\sqrt{A_a A_b}$, where A_a and A_b are the surface areas of the two spheres with radii a and b respectively and d is their separation distance. [7]

Q6.

(a) Compare "paramagnetic materials" and "ferromagnetic materials". [6]

(b) A coaxial cable has core of radius a and sheath of radius b and thickness t. A current I flows along the core, uniformly distributed across it, and returns along the sheath, uniformly distributed around it. Find the magnetic field for $r < a, a \le r \le b, b \le r \le b + t$ and $r > b + t$. [14]

Section C

Select **one** question from this section. (30 Marks)

Q7.

(a) State Maxwell's equations in integral form. [4]

(b) Interpret the physical meaning of the first two Maxwell's equations. [6]

(c) The electric field intensity of an electromagnetic wave in free space is given by $\boldsymbol{E}_y = 0, \boldsymbol{E}_z = 0$ and $\boldsymbol{E}_x = E_0 \cos\omega\left(t - \frac{z}{v}\right)$.

i. Determine the expression for the components of the magnetic field intensity $\boldsymbol{H}$. [15]

ii. Find ${E_x}/{H_y}$. [5]

Q8.

(a) Classify materials by their conductivity and compare their characteristics. [6]

(b) State the Poynting vector and its significance. [6]

(c) The electric field of a uniform plane wave propagating in the positive z-direction is given by $\boldsymbol{E} = E_0\cos(\omega t - \beta z)\boldsymbol{a}_x + E_0\sin(\omega t - \beta z)\boldsymbol{a}_y$ where E_0 is a constant. Find

i. the corresponding magnetic field [12]

ii. the Poynting vector [6]

Note:

1. In Spherical Coordinates $\nabla = \frac{\partial}{\partial r}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial\theta}\boldsymbol{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\boldsymbol{a}_\phi$
2. In Cylindrical Coordinates $\nabla = \frac{\partial}{\partial r}\boldsymbol{a}_r + \frac{1}{r}\frac{\partial}{\partial\phi}\boldsymbol{a}_\phi + \frac{\partial}{\partial z}\boldsymbol{a}_z$
3. $\varepsilon_0 = 8.854 \times 10^{-12} F/m$
4. $\mu_0 = 4\pi \times 10^{-7} N/A^2$