00228

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 03**
Name of the Examination	: Final Examination
Course Title and - Code	**: Differential Equations – ADU 3302/APU1142/ADE3302**
Academic Year	**: 2020/2021**
Date	**: 25.03.2022**
Time	**: 09.30 am – 11.30 am**
Duration	**: 02 hrs**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section **A** and Section **B**. Section **A** is compulsory and it consists of SIX Structured Essay Questions and carries 100 marks. You should answer in the space under each question.
3. Section B consists of **FIVE** essay type questions and answer only **THREE** of them. Each question in Section B carries 100 marks.
4. This paper consists of 06 pages.
5. Answer for each question in section B should commence from a new page.
6. Involvement in any activity that is considered as an exam offence will lead to punishment
7. Use a blue or black ink pen to answer the questions.
8. Clearly state your index number in your answer script.
9. At the end of the exam, attach Section A to the answer booklet and hand over to the supervisor.

SECTION A

1. **Answer all the questions in this section in the space provided.**

(a) Determine the order, degree and the independent variable of the following differential equation.

$$2\left(\frac{d^6 s}{dt^2}\right)^5 - 4\left(\frac{d^2 s}{dt^2}\right)^7 + 2s^2 = 2e^t$$

(b) Is $y = t^2$ a solution of $\frac{dy}{dt} = \frac{y}{t}$? Justify your answer.

(c) Find a solution to the initial-value problem $\frac{d^2y}{dx^2}+4y=0;\ y(0)=0,\ y'(0)=1,$ if the general solution to the differential equation is known to be $y(x)=c_1\sin 2x+c_2\cos 2x$ where c_1 and c_2 are arbitrary constants.

(d) Determine whether the differential equation $(y+\sin x)dx-(x-2y\cos x)dy=0$ is exact or not.

(e) Using a suitable substitution, transform the equation $y' + xy = xy^2$ into a first order linear differential equation.

(f) Find the general solutions of the homogeneous form of the differential equation $y'' - y = x$.

SECTION B

Answer **THREE** Questions **ONLY** from this section.

2. (a) Find the solutions of the differential equation $\frac{dy}{dx} = yx^2;\ y(0) = 1$.

(b) Determine whether the function $x + y\sin\left(\frac{y}{x}\right)^2$ is homogeneous or not and if so, find its degree.

(c) Consider the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$.

(i) Show that the above equation is homogeneous.

(ii) Using a suitable substitution, transform the given equation into a variable-separable form and find its solutions.

3. A body of mass m is thrown vertically upward into the air with an initial velocity v_0. The body encounters an air resistance proportional to its velocity.

(a) Find the equation of motion,

(b) Find an expression for the velocity of the body at any time t,

(c) Find an expression for the position of the body at any time t,

(d) the time at which the body reaches its maximum height.

4. (a) Consider the differential equation $\frac{d^2Q}{dt^2} - 3\frac{dQ}{dt} + 4Q = 0$.

(i) Find the characteristic equation of the above equation.

(ii) Using part (i) express the solutions of the given equation.

(b) Consider the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = x^2e^x$.

(i) Form the UC sets for the UC functions of the non-homogeneous term corresponds to the given equation. Generate the revised sets after performing, omitting and/or revising the sets.

(ii) Using the revised sets obtained in the part (b) (i), find the particular solution of the given equation.

(iii) Express the general solutions of the equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+4y=x^2e^x+e^x$ if the particular solution of the equation $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+4y=e^x$ is $\frac{1}{2}e^x$.

5. (a) Determine whether $-\frac{1}{x^2}$ is an integrating factor for the differential equation $ydx-xdy=0$.

(b) Determine a suitable integrating factor which makes the differential equation $\left(y^2-y\right)dx+xdy=0$ exact and then find the solutions.

(c) Using a suitable substitution transform the differential equation $\frac{dy}{dx}+xy=xy^3$ into a first order linear ordinary differential equation and then find its integrating factor.

6. (a) (i) Suppose $f(D)=a_0D^n+a_1D^{n-1}+\ldots+a_{n-1}D+a_n$, where $a_0,a_1,\ldots,a_n$ are constants. If k is a constant show that $f(D)e^{kx}=f(k)e^{kx}$.

(ii) Find the particular solution of the equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=e^{-x}+3$ using D-operator method.

(b) Consider the differential equation $\left(x^2-9\right)^2\frac{d^2y}{dx^2}+\left(x+3\right)\frac{dy}{dx}+2y=0$.
Find the singular points of the equation. Then identify the regular singular points and irregular singular points.