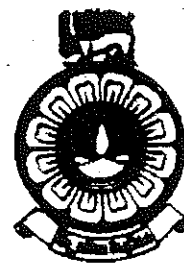


The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme



Department	: Mathematics
Level	: 03
Name of the Examination	: Final Examination
Course Title and - Code	: Differential Equations – ADU 3302/APU1142/ADE3302
Academic Year	: 2020/2021
Date	: 25.03.2022
Time	: 09.30 am – 11.30 am
Duration	: 02 hrs

General Instructions

1. Read all instructions carefully before answering the questions.
2. This paper consists of **TWO** sections, Section A and Section B. Section A is compulsory and it consists of SIX Structured Essay Questions and carries 100 marks. You should answer in the space under each question.
3. Section B consists of **FIVE** essay type questions and answer only **THREE** of them. Each question in Section B carries 100 marks.
4. This paper consists of 06 pages.
5. Answer for each question in section B should commence from a new page.
6. Involvement in any activity that is considered as an exam offence will lead to punishment
7. Use a blue or black ink pen to answer the questions.
8. Clearly state your index number in your answer script.
9. At the end of the exam, attach Section A to the answer booklet and hand over to the supervisor.

SECTION A

1. Answer all the questions in this section in the space provided.

(a) Determine the order, degree and the independent variable of the following differential equation.

$$2\left(\frac{d^6s}{dt^2}\right)^5 - 4\left(\frac{d^2s}{dt^2}\right)^7 + 2s^2 = 2e^t$$

(b) Is $y = t^2$ a solution of $\frac{dy}{dt} = \frac{y}{t}$? Justify your answer.

- (c) Find a solution to the initial-value problem $\frac{d^2y}{dx^2} + 4y = 0$; $y(0) = 0$, $y'(0) = 1$, if the general solution to the differential equation is known to be $y(x) = c_1 \sin 2x + c_2 \cos 2x$ where c_1 and c_2 are arbitrary constants.

- (d) Determine whether the differential equation $(y + \sin x)dx - (x - 2y \cos x)dy = 0$ is exact or not.

(e) Using a suitable substitution, transform the equation $y' + xy = xy^2$ into a first order linear differential equation.

(f) Find the general solutions of the homogeneous form of the differential equation $y'' - y = x$.

SECTION B

Answer **THREE** Questions **ONLY** from this section.

2. (a) Find the solutions of the differential equation $\frac{dy}{dx} = yx^2$; $y(0) = 1$.
- (b) Determine whether the function $x + y \sin\left(\frac{y}{x}\right)^2$ is homogeneous or not and if so, find its degree.
- (c) Consider the differential equation $\frac{dy}{dx} = \frac{x^2 + y^2}{xy}$.
- (i) Show that the above equation is homogeneous.
- (ii) Using a suitable substitution, transform the given equation into a variable-separable form and find its solutions.
3. A body of mass m is thrown vertically upward into the air with an initial velocity v_0 . The body encounters an air resistance proportional to its velocity.
- (a) Find the equation of motion,
- (b) Find an expression for the velocity of the body at any time t ,
- (c) Find an expression for the position of the body at any time t ,
- (d) the time at which the body reaches its maximum height.
4. (a) Consider the differential equation $\frac{d^2Q}{dt^2} - 3\frac{dQ}{dt} + 4Q = 0$.
- (i) Find the characteristic equation of the above equation.
- (ii) Using part (i) express the solutions of the given equation.
- (b) Consider the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = x^2e^x$.
- (i) Form the UC sets for the UC functions of the non-homogeneous term corresponds to the given equation. Generate the revised sets after performing, omitting and/or revising the sets.
- (ii) Using the revised sets obtained in the part (b) (i), find the particular solution of the given equation.

(iii) Express the general solutions of the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = x^2e^x + e^x$ if the particular solution of the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 4y = e^x$ is $\frac{1}{2}e^x$.

5. (a) Determine whether $-\frac{1}{x^2}$ is an integrating factor for the differential equation $ydx - xdy = 0$.

(b) Determine a suitable integrating factor which makes the differential equation $(y^2 - y)dx + xdy = 0$ exact and then find the solutions.

(c) Using a suitable substitution transform the differential equation $\frac{dy}{dx} + xy = xy^3$ into a first order linear ordinary differential equation and then find its integrating factor.

6. (a) (i) Suppose $f(D) = a_0D^n + a_1D^{n-1} + \dots + a_{n-1}D + a_n$, where a_0, a_1, \dots, a_n are constants. If k is a constant show that $f(D)e^{kx} = f(k)e^{kx}$.

(ii) Find the particular solution of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{-x} + 3$ using D-operator method.

(b) Consider the differential equation $(x^2 - 9)^2 \frac{d^2y}{dx^2} + (x + 3) \frac{dy}{dx} + 2y = 0$.

Find the singular points of the equation. Then identify the regular singular points and irregular singular points.