00270

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: **Mathematics**
Level	: **Level 03**
Name of the Examination	: **Final Examination**
Course Code and Title	: **PUU1141/PUE3141/ PEU3301 Foundations of Mathematics**
Academic Year	: **2020/ 2021**
Date	: **15.03.2022**
Time	: **09.30 a.m.-11.30 a.m.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **six (06)** questions in **two (02)** pages.
3. Answer any **four (04)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
6. Use blue or black ink to answer the questions.
7. Circle the number of the questions you answered in the front cover of your answer script.
8. Clearly state your index number in your answer script.

Q1

(a) Solve the Solve the inequality $\frac{|x-1|}{|3x+1|} > 1$, where $x \in \mathbb{R}$.

(b) Let $f: X \to Y$ be a function and A, B subsets of Y. Prove the identity $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$ by showing that the set in either side of the equal sign is a subset of the other.

(c) Let $f: \mathbb{Z} \to \mathbb{Z}$ be defined by $f(n) = n^2$. Show that if $A = \{0, 1, 2, 3\}$ and $B = \{-2, -1, 0, 1\}$ then $f(A \cap B) \neq f(A) \cap f(B)$.

Q2

(a) Let R be the relation defined on $\mathbb{Z}$ by aRb if and only if $3a + b$ is a multiple of 4.

(i) Prove that R is an equivalence relation on $\mathbb{Z}$.

(ii) Find the equivalence class of 0.

(iii) Find the equivalence class of 2.

(b) Let $a, b \in \mathbb{Z} \setminus \{0\}$. Prove that there exist $x, y \in \mathbb{Z}$ such that $ax + by = \gcd(a, b)$, where $\gcd(a, b)$ denotes the greatest common divisor a and b.

Q3

(a) Prove that for $a, b \in \mathbb{R}$ is such that $a < b$,

$$\bigcap_{n=1}^{+\infty} \left(a - \frac{1}{n}, b + \frac{1}{n}\right) = [a, b].$$

(b) Prove that $\sqrt{6}$ is an irrational number.

Q4

(a) Let $A_1, A_2, \cdots, A_n$ and B be sets. By using the Principle of Mathematical Induction, prove that for each $n \in \mathbb{N}$,

$$(A_1 - B) \cap (A_2 - B) \cap \cdots \cap (A_n - B) = (A_1 \cap A_2 \cdots \cap A_n) - B.$$

(b) Let the function $f: (a, b) \to (c, d)$ be defined by $f(x) = \left(\frac{d-c}{b-a}\right)(x - a) + c$. Show that f is a bijection.

Q5

(a) Let $a_n = \frac{3}{n^2+3n+2}$ for $n \in \mathbb{N}$.

(i) Find A and B such that $a_n = \frac{A}{(n+1)} + \frac{B}{(n+2)}$

(ii) Using the above result, show that the nth partial sum

$$S_n = \sum_{k=1}^{n} \frac{3}{k^2+3k+2} = \frac{3}{2} - \frac{3}{n+2}, \text{ for each } n \in \mathbb{N}.$$

(iii) Show that the series $\sum_{n=1}^{\infty} a_n$ converges. Find $\sum_{n=1}^{\infty} a_n$.

(b) By using the definition for limit of a sequence, prove that

$$\lim_{n \to +\infty} \frac{2n+5}{7n+8} = \frac{2}{7}.$$

Q6

(a) Let f, g and h be the functions given by

$$f(x) = \frac{1}{x+1}, x \in \mathbb{R} \setminus \{1\}, \; g(x) = \frac{\sqrt{x-1}}{x}, x \in [1, +\infty) \text{ and}$$

$$h(x) = x^2 + 1, x \in \mathbb{R}.$$

Find $f \circ g \circ h$ and its domain.

(b) Let $S = \left\{\frac{2n+1}{n+1} : n \in \mathbb{N}\right\}$.

(i) Prove that the set S is bounded.

(ii) Deduce that both $\sup S$ and $\inf S$ exist.

(iii) Find $\sup S$ and $\inf S$. Justify your answers.

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