00226

The Open University of Sri Lanka

Faculty of Natural Sciences

B.Sc./ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Newtonian Mechanics I – ADU4301**
Academic Year	**: 2020/21**
Date	**: 21.03.2022**
Time	**: 1.30 p.m. To 3.30. p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (4) pages.
3. Answer any **FOUR (4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1. A particle of mass m is projected vertically upwards with a velocity U from a point on horizontal ground. The particle is subjected to air resistance of magnitude mkv^2 where v is the velocity of the particle and k is a positive constant.

   (a) Find the greatest height attained by the particle.
   (b) Find also the velocity when it will return to the point of projection.

2. (a) With the usual notation, show that in intrinsic coordinates, the velocity and acceleration $\underline{a}$ of a particle moving in a plane curve are given by

   $\underline{v} = \dot{s}\underline{t}$ and $\underline{a} = \ddot{s}\underline{t} + \frac{\dot{s}^2}{\rho}\underline{n}$ respectively.

   (b) A smooth wire in the form of an arch of the cycloid, with intrinsic equation $s = 4a\sin\psi$, is fixed in a vertical plane with its vertex downwards. The tangent at the vertex is horizontal. A particle is projected at time $t = 0$ with velocity V from the cusp of the cycloid down the arc. Show that at time t,

   $$\left(\frac{ds}{dt}\right)^2 = \frac{g}{4a}(16a^2 - s^2) + V^2.$$

   Hence, show that the time of reaching the vertex is $2\sqrt{\frac{g}{a}}\, tan^{-1}\left(\sqrt{\frac{4ag}{V}}\right)$.

3. (a) With the usual notation, show that, in plane polar coordinates, the velocity $\underline{v}$ and acceleration $\underline{a}$ of a particle moving in a plane are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$

   and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$ respectively.

   (b) A particle is at rest on a smooth horizontal plane, which commences to turn about a straight line lying in that plane with constant angular velocity ω downwards. If a is the distance of the particle from the axis of rotation at time $t = 0$, then show that, at time t

$$r(t) = \cosh \omega t + \frac{g}{2\omega^2} \sinh \omega t - \frac{g}{2\omega^2} \sin \omega t$$

Show also that the particle will leave the plane at time t given by the equation $a \sinh \omega t + \frac{g}{2\omega^2} \cosh \omega t = \frac{g}{2\omega^2} \cos \omega t$.

4. (a) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} - \underline{u}\frac{dm}{dt}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, the matter being added at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(b) A rocket uses fuel at a constant rate λ. The rocket moves forwards by ejecting used fuel backwards from the rocket with speed u relative to the rocket. At time t rocket is moving with speed v and the combined mass of the rocket and its fuel is m. The rocket starts from rest at time $t = 0$ with a total mass M.

Show that

(i) $m\frac{dv}{dt} = \lambda u$, and

(ii) $v = u\ln\left(\frac{M}{M-\lambda t}\right)$.

Also find an expression for the distance travelled at time t.

5. (a) With the usual notation show that the equation of the central orbit of a particle moving in a plane is given by

$$\frac{d^2u}{d\theta^2} + u = \frac{F}{h^2u^2} \qquad \text{and} \qquad \dot{\theta} = hu^2.$$

(b) A particle P moves in a path with polar equation $r = \dfrac{2a}{2+\cos\theta}$, coordinates being measured with respect to a pole O and initial line OA. Given that at any time t during the motion $r^2\dot{\theta} = h$ (constant), determine the central force.

6. A uniform circular disc, of mass m and radius r, has a diameter AB. The point C on AB is such that $AC = r/2$. The disc can rotate freely in a vertical plane about a horizontal axis through C, perpendicular to the plane of the disc. The disc makes small oscillations in a vertical plane about the position of equilibrium in which B is below A.

(a) Show that the motion is approximately simple harmonic.

(b) Show that the period of this approximately simple harmonic motion is $\pi\sqrt{(6r/g)}$.

(c) The speed of B when it is vertically below A is $\sqrt{(gr/54)}$. The disc comes to instantaneous rest when CB makes an angle α with the downward vertical. Find an approximate value of α.