00309

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 04
Name of the Examination	: Final Examination
Course Code and Title	: ADU4303 Applied Linear Algebra & Differential Equations
Academic Year	: 2020/2021
Date	: 16.03.2022
Time	: 09.30a.m.-11.30a.m.
Duration	: 2 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of 06 questions in 04 pages.
3. Answer any 04 questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary.
5. Relevant log tables are provided where necessary.
6. Having any unauthorized documents/ mobile phones in your possession is a punishable offense.
7. Use blue or black ink to answer the questions.
8. Circle the number of the questions you answered in the front cover of your answer script.
9. Clearly state your index number in your answer script.

1. (a) Show that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (4-x)^2(5x+4)$.

(b) Prove that the product of the two matrices

$$\begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \text{ and } \begin{pmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{pmatrix}$$

is zero when θ and ϕ differ by an odd multiple of $\frac{\pi}{2}$.

(c) Explain the consistency or the inconsistency of a system of m linear equations in n unknowns.

For which rational numbers a, b and c does the follwing system have

(i) No solution.

(ii) A unique solution & find the Solution if $a \neq b$.

(iii) For infinitely many solutions, if $c \neq d$, show that $a+b = c+d$.

$$x + y + z = 1$$

$$ax + by + cz = d$$

$$a^2 x + b^2 y + c^2 z = d^2$$

2. (a) Find the orthogonal transformation which transforms the quadratic form

$10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_1x_3 - 4x_2x_1$ to a canonical form.

(b) Determine for what values of the numbers a and b, $C = aA + bB$ is Skew-Hermitian given that A and B Skew-Hermitian.

(c) Show that the following matrix is unitary:

$$\begin{pmatrix} \frac{1}{2}(1+i) & \frac{1}{2}(-1+i) \\ \frac{1}{2}(1+i) & \frac{1}{2}(1-i) \end{pmatrix}$$

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3. Find the general solution of each of the systems of simultaneous differential equations, given below in the standard notation:

(a) $\dot{x}_1 = x_1 + x_2 - x_3$
$\dot{x}_2 = 2x_1 + 3x_2 - 4x_3$
$\dot{x}_3 = 4x_1 + x_2 - 4x_3$,

(b) $\dot{x}_1 = 2x_1 + 3x_2 + 4e^{3t}$
$\dot{x}_2 = -x_1 - 2x_2 - e^{3t}$

(c) $\ddot{x} = 8x - 5y$
$\ddot{y} = 10x - 7y$

4. (a) Find a sinusoidal particular solution for the following system of partial differential equations:

$$\ddot{x}_1 + 2\ddot{x}_2 + \dot{x}_1 + x_1 - 3x_2 = \sin t$$
$$3\ddot{x}_1 + \ddot{x}_2 + 2\dot{x}_2 + 2x_1 + x_2 = \cos t - 2\sin t.$$

(b) Use the change of variable $x = \cos t$ $(0 < t < \pi)$ to find the general solution of the differential equation

$(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - \lambda y = 0$, $(-1 < x < 1)$ where λ is a positive constant.

(c) If the density ρ is a function of x and y, and the transformation $\zeta = x^2 - y^2$, $\phi = 2xy$, is made, show that the first order partial differential equation $x\dfrac{\partial \rho}{\partial x} - y\dfrac{\partial \rho}{\partial y} = 0$ becomes $\dfrac{\partial \rho}{\partial \zeta} = 0$, provided $x^2 + y^2 \neq 0$. Hence find ρ as a function of x and y.

5. (a) Find the general solution of the following pair of partial differential equations:

$$\frac{\partial u}{\partial x} = \frac{2x}{x^2 - y^2} + 4x(x-y) + 2(x-y)^2$$
$$\frac{\partial u}{\partial y} = \frac{-2y}{x^2 - y^2} - 4x^2 + 4xy + 3y^2$$

(b) Find the general solution of the following partial differential equation by using the integrating factor method. (u is a function of the two variables x and y.)

$$\frac{\partial u}{\partial x}+\left(\frac{2xy+1}{x}\right)u=e^{-2xy}, \quad (x\neq 0)$$

(c) Applying the change of variables $\zeta=x^2-y$ and $\phi=x^2+y$ to the equation $\frac{\partial^2 u}{\partial x^2}-\frac{1}{x}\frac{\partial u}{\partial x}-4x^2\frac{\partial^2 u}{\partial y^2}=0 \quad (x\neq 0)$, verify that the general solution $u=f(x^2-y)+g(x^2+y)$ of the above equation also satisfies the equation $\frac{\partial^2 u}{\partial \zeta \partial \phi}=0$.

6. (a) Find the equations of the characteristic curves for the partial differential equation

$$\frac{\partial u}{\partial x}-3\frac{\partial u}{\partial y}=u$$

and hence find the general solution.

If $u(x, y)=y$ on $x=0$, what is the solution for the given partial differential equation?

(b) Solve the following partial differential equation:

$$8\frac{\partial^2 u}{\partial x^2}-2\frac{\partial^2 u}{\partial x \partial y}-3\frac{\partial^2 u}{\partial y^2}=0.$$