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00275

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 04**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Statistical Distribution Theory – ADU4300/ADE4300**
Academic Year	**: 2020/2021**
Date	**: 17.12.2021**
Time	**: 2.00 p.m. – 4. p.m.**
Duration	**: 02 hrs**

General Instructions

1. Read all the instructions carefully before answering the questions.
2. Answer only 4 questions out of 6 questions.
3. Non programmable calculators are permitted. Statistical tables are provided.
4. This paper consists of 05 pages.
5. Involvement in any activity that is considered as an exam offence will lead to punishment.
7. Use a blue or black ink pen to answer the questions.
8. Clearly state your index number in your answer script.
9. At the end of the exam, and hand over the answer script to the supervisor.

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B.Sc/B.Ed. Degree, Continuing Education Programme

Final Examination - 2020/2021

Level 04 Applied Mathematics

ADU4300/ADE4300– Statistical Distribution Theory

Duration: - Two Hours

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Instructions

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1.

The time to failure of an electric bulb (***X***) in years is well modeled by the following probability distribution.

$$f(x) = ke^{-kx} \quad ; x > 0$$

Suppose the median time to failure of an electric bulb is 1.38 years

(i) Fine the value of k. Round your answer to nearest two decimal places.

(ii) Find the expected time to failure of a randomly selected electric bulb.

(iii) If the manufacturer offers a one year warranty period, what percentages of the bulbs will fail during the warranty period?

(iv) Find the cumulative distribution function of the time to failure of a randomly selected bulb.

(v) Determine the probability that a bulb fails in the interval from 1 to 3 years.

2.

Applicants seeking admission to a Graduate School of Business requires to take an Entrance Examination. Scores of the Entrance Examination ***(X)*** are roughly normally distributed with a mean of 40 and a standard deviation of 10.

(i) What is the probability that a randomly selected individual scoring above 60 in the Entrance Examination?

(ii) What is the lowest score of 5% of the top scorers?

(iii) What percentage of students will score marks in between 20 to 60?

(iv) Suppose a random sample of marks of five students were selected. Let sample mean = $\bar{X}$

a. What is the distribution of the sample mean ($\bar{X}$)?

b. Find the $\Pr(30 < \bar{X} < 50)$

3.

a. Suppose that a quiz consists of n True-False questions. A student hasn't studied for the exam and will just randomly guess the answers to each question.

(i) If $n = 10$, find the probability that the student will get 7 or fewer answers correct?

(ii) If $n = 20$, find the probability that the student will get 14 or fewer answers correct, using a suitable approximation?

(iii) Suppose that the questions are released one by one and answer will be marked as correct or incorrect immediately after the attempt to the question. Let $n = 10$.

I. What is the portability that first correct answer occurred at the third question?

II. What is the portability that third correct answer occurred at the sixth question?

b. Suppose the average number of customers at an ATM in 10 -minute interval is 3 on a weekday. What is the probability that there will be more than 4 customers in 30 minutes interval at the ATM on a Monday?

4. Suppose that X_1, X_2, X_3, X_4 are independent random variables described as

$$X_1 \sim N(3,4) \qquad X_2 \sim N(5,4) \qquad X_3 \sim exp(2) \qquad X_4 \sim Gamma(2,2) \qquad X_5 \sim Bin(6,0.4)$$

$$X_6 \sim Bin(4,0.4) \qquad X_7 \sim Poisson(2) \qquad X_8 \sim Poisson(3)$$

Find the following probabilities. Show your calculations and state the justifications clearly.

(i) $\Pr[21 < (2X_1 + 3X_2) < 25)]$

(ii) $\Pr[(X_3 + X_4) > 2)]$

(iii) $\Pr[(X_5 + X_6) > 6)]$

(iv) $\Pr[(2X_7 + X_8) > 5)]$

5.

The joint distribution of random variables X and Y is given below.

$P(X=x, Y=y)$		X			
		0	1	2	3
Y	1	0.1	0.1	0.08	0
	2	0.01	k	0	0.05
	3	0.19	0.1	0.3	0.02

(i) Find the value of k.

(ii) Calculate the following probabilities.

(I) $Pr(X<2, Y=2)$ (II) $Pr(X=3)$ (iii) $Pr(Y<2)$

(iii) Find the marginal distribution of X and marginal distribution of Y.

(iv) A student says that "the random variables X and Y are independent". State whether the students' statement is true or false. Justify your answer.

(v) Calculate $Pr\,(X = 1|Y = 3)$.

(vi) Calculate $E\,(X|Y = 3)$.

6.

a.

Random variable X has the probability mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \quad ; \quad x = 0,1,2,3, \ldots \ldots \ldots$$

(i) Let $M_x(t)$ be the moment generating function of X. Show that $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i) find the expected value of X and variance of X

b.

Let X be a random variable. The variance of the random variable X is defines as

$V(X) = E\{[X - E(X)]^2\}$.

(i) Show that $V(X) = \{E(X^2) - [E(X)]^2\}$

(ii) $V(aX) = a^2V(X)$

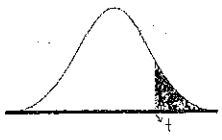
Left tail values of Standard Gamma Table

W - gamma(α,1) - This table contain the probabilities Pr(W ≤ w)

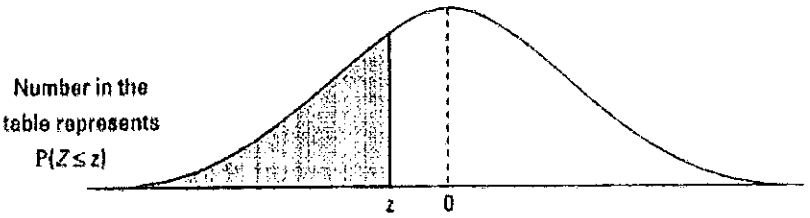
	α					
w	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432

TABLE B: t-DISTRIBUTION CRITICAL VALUES

	Tail probability p									
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153



Normal Distribution Table



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
−3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641