(20)
00118

The Open University of Sri Lanka
Faculty of Natural Sciences
BSc Degree Programme

Department	: Physics
Level	: 05
Name of the Examination	: Final Examination
Course Title and - Code	: SOLID STATE PHYSICS – PHU5312
Academic Year	: 2020/2021
Date	: 18.12.2021
Time	: 1.30 p.m. – 3.30 p.m.
Duration	: 02 hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (06) questions in (05) pages.
3. **Answer any Four (04) questions only**. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. **Clearly state your index number** in your answer script

1) (i) What is the difference between crystalline and non-crystalline (amorphous) solids? **(02 marks)**

(ii) Explain the terms of

(b) lattice,

(c) basis and

(d) crystal structure. **(03 marks)**

(iii) Name the three simple crystal structures in which most common metals exist. **(03 marks)**

(iv) How many numbers of lattice points are there in a fcc unit cell? **(02 marks)**

(v) Draw the following planes for a cubic unit cell.

(a) $(1\,\bar{2}\,1)$ (b) $(0\,0\,1)$ (c) $(1\,1\,2)$ **(06 marks)**

(vi) A lattice has primitive translation vectors

$$\underset{\sim}{a} = a\hat{i},$$

$$\underset{\sim}{b} = a\hat{i} + 2a\hat{j} \text{ and}$$

$$\underset{\sim}{c} = 2a\hat{j} + a\hat{k},$$

where $\hat{i}, \hat{j}, \hat{k}$ are the usual Cartesian unit vectors, and a is a constant.

Determine the reciprocal lattice vectors. **(09 marks)**

2) (i) Show that for any cubic lattice the separation of the planes corresponding to Miller indices (hkl) is given by: $d_{hkl} = \dfrac{a}{\sqrt{h^2+k^2+l^2}}$, where a is the lattice parameter.

(5 Marks)

(ii) Briefly describe the Bragg's diffraction in crystals and show that the Bragg condition for crystal diffraction on (hkl) planes is given by:

$$2d_{hkl}\sin\theta_{hkl} = n\lambda$$

where the symbols have their usual meanings. **(5 Marks)**

(iii) The Bragg angle for reflection from (110) plane in bcc Iron is 22^0 for an X-ray of wavelength $\lambda = 1.54$Å.

a) Compute the cube edge for Iron

b) Determine the Bragg angle for the reflection from (111) planes.

c) Calculate the density of bcc iron

The atomic weight of iron is 55.8 and Avogadro's number $N_A = 6.02 \times 10^{23}$ mol^{-1}

(15 marks)

3) According to the Lennard-Jones pair potential model, the cohesive energy per atom in a crystal can be expressed as:

$$U(r) = 2\varepsilon\left[A\left(\frac{\sigma}{r}\right)^{12} - B\left(\frac{\sigma}{r}\right)^{6}\right],$$

where r is the nearest neighbour separation, σ and ε are constants. A and B are known as lattice sums and are determined by the type of crystal structure.

(a) Sketch the variation of the cohesive energy with r, marking on your plot the equilibrium separation and the cohesive energy at this point.

(6 marks)

(b) Show that, at equilibrium, the nearest neighbour separation (r_0) is given by

$$r_0 = \sigma\left(\frac{2A}{B}\right)^{\frac{1}{6}}.$$

(6 marks)

(c) Show that the equilibrium cohesive energy (U_0) is given by

$$U_0 = -\varepsilon\left(\frac{B^2}{2A}\right).$$

(6 marks)

(d) A face centred cubic (fcc) crystal has the values $A = 12.12$ and $B = 14.45$. For a body centred cubic (bcc) structure, $A = 9.11$ and $B = 12.25$. Show that the cohesive energy of an fcc structure is 4% lower than that of a bcc structure. Comment on the physical significance of this result.

(7 marks)

4) (a) Show that for a one-dimensional linear chain of identical atoms having mass m, the dispersion relation for the longitudinal vibrations is given by

$$\omega = 2\sqrt{\frac{\beta}{m}}\sin\left(\frac{ka}{2}\right)$$

Where ω and k are respectively the angular frequency and wave vector of the longitudinal phonon wave in the linear atomic chain and β is the binding force per unit length between the adjacent atoms separated by distance a.

(10 Marks)

(b) Show that the group velocity of sound in the long-wavelength limit is

$$v_g = a\left(\frac{\beta}{m}\right)^{1/2}$$

(7 Marks)

(c) The one-dimensional chain consists of copper atoms. The speed of sound in copper is 6420 m s^{-1}, and a = 0.405 nm. Determine the angular frequency ω of the wave that propagates along the chain.

(8 Marks)

5) The Einstein theory of specific heats gives the following equation for the molar specific heat of a solid at temperature T:

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \frac{\exp\left(\frac{\theta_E}{T}\right)}{\left[\exp\left(\frac{\theta_E}{T}\right)-1\right]^2}$$, the Einstein temperature θ_E is given by $\frac{\hbar\omega_E}{k_B}$

(a) Show that, in the high temperature limit, C_V approaches the classical value $3R$.

(5 Marks)

(b) Show that in the low temperature limit, C_V approaches the value

$$C_V = 3R\left(\frac{\theta_E}{T}\right)^2 \exp\left(-\frac{\theta_E}{T}\right).$$

(5 Marks)

(c) If the Einstein temperature for copper is 1990 K, then, show that the Einstein frequency of vibration for copper is within the infrared region of the electromagnetic spectrum.

(5 Marks)

(d) The Debye theory of specific heats at low temperatures gives the following equation for the molar specific heat of a solid at temperature T:

$$C_V = \frac{12}{5}\pi^4 R\left(\frac{T}{\theta_D}\right)^3, \text{ Debye temperature } \theta_D \text{ is given by } \frac{\hbar\omega_D}{k_B}$$

If the Debye temperature for diamond is 2230, determine

(i) the highest possible vibrational frequency, and

(ii) the molar heat capacity

of diamond at temperature 10 K **(10 Marks)**

6) (a) According to Fermi-Dirac statistics, the probability that an electron has an energy E at a temperature T is given by the distribution function

$$f(E) = \frac{1}{e^{\frac{E-E_F}{k_B T}} + 1}$$

where k_B is the Boltzmann constant and E_F is the Fermi energy.

(i) Sketch the form of the Fermi-Dirac distribution function $f(E)$ for the electron gas at $T = 0$ K and $T > 0$ K. Indicate on your plot the location of the Fermi energy E_F.

(4 Marks)

(ii) At what temperature can we expect a 10% probability that the electrons in silver have an energy which is 1% above the Fermi energy? Fermi energy of silver 5.5 eV.

(4 Marks)

(b) Assuming that the density of states is given by $g(E)dE = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} E^{1/2} dE$ and the Fermi energy is given by $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$ where n is the number of electrons per unit volume and if the average energy of a free electron can be written as

$$\langle E \rangle = \frac{1}{N}\int_0^{E_F} E\, g(E)\, dE, \text{ where } N \text{ is the total number of free electrons.}$$

Show that (at 0 K) the average energy of an electron in a metal is 60% of the Fermi energy (E_F). **(8 Marks)**

(c) The density of Zn is 7.3×10^3 kg m^{-3} and its atomic weight is 65.4. Note that the Zn is a divalent metal and its effective mass of the electron is $0.85 m_e$. Determine

(i) the number of electrons per unit volume

(ii) the Fermi energy in Zn.

(iii) average energy at 0 K. **(9 Marks)**

****

Physical Constants

electron charge	$e = 1.60\times10^{-19}$ C
electron mass	$m_e = 9.11\times10^{-31}$ kg $= 0.511$ MeV c^{-2}
proton mass	$m_p = 1.673\times10^{-27}$ kg $= 938.3$ MeV c^{-2}
neutron mass	$m_n = 1.675\times10^{-27}$ kg $= 939.6$ MeV c^{-2}
Planck's constant	$h = 6.63\times10^{-34}$ J s
Dirac's constant ($\hbar = h/2\pi$)	$\hbar = 1.05\times10^{-34}$ J s
Boltzmann's constant	$k_B = 1.38\times10^{-23}$ J K^{-1} $= 8.62\times10^{-5}$ eV K^{-1}
speed of light in free space	$c = 299\,792\,458$ m s^{-1} $\approx 3.00\times10^{8}$ m s^{-1}
permittivity of free space	$\varepsilon_0 = 8.85\times10^{-12}$ F m^{-1}
permeability of free space	$\mu_0 = 4\pi\times10^{-7}$ H m^{-1}
Avogadro's constant	$N_A = 6.02\times10^{23}$ mol^{-1}
gas constant	$R = 8.32$ J mol^{-1} K^{-1}
ideal gas volume (STP)	$V_0 = 22.4$ l mol^{-1}
gravitational constant	$G = 6.67\times10^{-11}$ N m^2 kg^{-2}
Rydberg constant	$R_\infty = 1.10\times10^{7}$ m^{-1}
Rydberg energy of hydrogen	$R_H = 13.6$ eV
Bohr radius	$a_0 = 0.529\times10^{-10}$ m
Bohr magneton	$\mu_B = 9.27\times10^{-24}$ J T^{-1}
fine structure constant	$\alpha \approx 1/137$
Wien displacement law constant	$b = 2.898\times10^{-3}$ m K
Stefan's constant	$\sigma = 5.67\times10^{-8}$ W m^{-2} K^{-4}
radiation density constant	$a = 7.55\times10^{-16}$ J m^{-3} K^{-4}
mass of the Sun	$M_\odot = 1.99\times10^{30}$ kg
radius of the Sun	$R_\odot = 6.96\times10^{8}$ m
luminosity of the Sun	$L_\odot = 3.85\times10^{26}$ W
mass of the Earth	$M_\oplus = 6.0\times10^{24}$ kg
radius of the Earth	$R_\oplus = 6.4\times10^{6}$ m

Conversion Factors

1 u (atomic mass unit) $= 1.66\times10^{-27}$ kg $= 931.5$ MeV c^{-2}	1 Å (angstrom) $= 10^{-10}$ m
1 astronomical unit $= 1.50\times10^{11}$ m	1 g (gravity) $= 9.81$ m s^{-2}
1 eV $= 1.60\times10^{-19}$ J	1 parsec $= 3.08\times10^{16}$ m
1 atmosphere $= 1.01\times10^{5}$ Pa	1 year $= 3.16\times10^{7}$ s

****