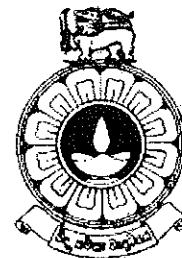


(25)
00243

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme



Department	: Computer Science
Level	: 05
Name of the Examination	: Final Examination (1st Semester)
Course Title and - Code	: CSU 5304 Mathematics for Computing
Academic Year	: 2020/2021
Date	: 08.12.2021
Time	: 9.30 am -11.30 am
Duration	: Two hours only

General Instructions

1. Read all instructions carefully before answering the questions.
 - 2: This question paper consists of **(06)** questions in **(05)** pages.
 3. Answer any ... **(04)** questions only. All questions carry equal marks.
 4. Answer for each question should commence from a new page.
 5. Draw fully labelled diagrams where necessary
 6. Involvement in any activity that is considered as an exam offense will lead to punishment
 7. Use blue or black ink to answer the questions.
 8. Clearly state your index number in your answer script
-

The Open University of Sri Lanka
 B.Sc. Degree Programme – Level 05
 Department of Computer Science
 CSU 5304 – Mathematics for Computing
 Final Examination 2020/2021
 Duration: Two hours only (2 hours)



Date: 8th December 2021

Time: 9.30 a.m. – 11.30 a.m.

ANSWER FOUR QUESTIONS ONLY

(Q1)

- (i) Write down the conditions to be satisfied in order to perform the following matrix operations.
- Addition of two matrices
 - Multiplication of two matrices
 - Finding the determinant of a matrix
- (ii) The two matrices A and B are defined by:

$$A = \begin{bmatrix} -2 & 1 & 2 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 2 \\ k+3 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{Where } k \in \mathbb{R}$$

- Obtain the matrix $2A^T - B$; where A^T is the transpose of A.
- (i) Find and simplify an expression for the determinant of B.
 (ii) Find the value of k such that B^{-1} does not exist.

(iii) For the matrix $C = \begin{bmatrix} 2 & 2\alpha - \beta & -1 \\ 3\alpha - 2\beta & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$

Obtain the values of α and β such that

$$C^T = \begin{bmatrix} 2 & -5 & -1 \\ -1 & 4 & 3 \\ -1 & 3 & 2 \end{bmatrix} \quad \text{where } C^T \text{ is the transpose of matrix } C.$$

(iv)

- a) Considering any two square matrices, write the commutative law for matrices under addition. (Hint: you can consider the two matrices as P and Q).
- b) Show that if D is any square matrix, then (D+I) and (D-I) commute. Where I is the identity matrix with same order as D.
- c) If (D+I) is a non-singular matrix, use the above condition in part (b) to show that, $(D+I)^{-1}$ and (D-I) commute.

(25 marks)

(Q2)

- (i) (a) Write the general definition of "Induction"
- (b) A similar concept is used in Mathematics known as the Principal of Mathematical Induction. When can the Principal of Mathematical Induction be used?
- (c) Write the steps that you would follow in a proof, when using the Principal of Mathematical Induction.

(ii) Use the principle of Mathematical Induction for the following proofs.

- a) Prove that for all
- $n \geq 1$

$$1+4+7+\dots+(3n-2) = \frac{n(3n-1)}{2}$$

- b) Verify that for any positive integer n,
- (6^n-1)
- is divisible by 5.

(iii) Select the correct answer and give the justification for your answer.

- a) Let P(x) denote the statement
- $x > 7$
- . Which of these have truth value "true" ?.

- P(0)
- P(-4)
- P(6)
- P(9)

- b) Let Q(x) be the statement
- $x < 5$
- . What is the truth value of
- $\forall x Q(x)$
- having domain as real numbers?

- True
- False

- c) Let R(x) be the statement
- $x > 2$
- . What is the truth value of the quantification
- $\exists x R(x)$
- having domain as real numbers?

- True
- False

(25 marks)

(Q3)

- (i) Write the standard symbols for the following sets.
- Set of Natural numbers
 - Set of Rational numbers
 - Set of Real numbers
- (ii) Write the definitions of the following sets. **(Marks will not be given, if you take examples and explain).**
- Equal sets
 - Equivalence set
 - Power set
 - Proper subset
- (iii) There are 35 students in an Art class and 57 students in a Music class. Find the number of students who are either in the Art class or in the Music class according to the conditions (a) and (b).
- When two classes meet at different hours and 12 students are enrolled in both activities.
 - When two classes meet at the same hour.
- (iv) (a) Write the names of four intervals, that you studied under the course CSU 5304.
- (b) Give a graphical representation for the interval $\{x: x < -2 \text{ or } x > 3\}$.
- (c) Consider the inequality $10 < (-3a + 10) \leq 34$. Solve and describe your answer in both inequality notation and interval notation.
- (d) Find the Sum of all the integer values of x that satisfy the following inequality.

$$|4 - 2x| < 5$$

(25 marks)

(Q4)

- (i) (a) What is meant by "relation" in regards to mathematics?
- (b) Is a function a relation? Justify your answer.
- (c) What are the main component/components to form a function?
- (ii) Given that $A = \{5, 6, 7, 8, 9, 10\}$ and $B = \{7, 8, 9, 10, 11, 13\}$
- Define a relation R from set A to set B .
 - Draw a diagram to show the relationship.
 - Is the relationship a function? Justify your answer.
- (iii) If two functions are given by $f(x) = 5 - 4x$; $x \in \mathbb{R}$ and $g(x) = x^2 - 3x$; $x \in \mathbb{R}$
- Find $f(3x)$ in terms of x .

- (b) Find $f(f(x))$ in its simplest form.
- (c) Find the inverse of $f(x)$.
- (d) Solve the equation $gf(x)=0$

(25 marks)

(Q5)

- (i) (a) If p and q are two statements, the implication is defined by $p \rightarrow q$ (if p then q).
Give three names that are related with this implication.
- (b) An implication is given as:
"If it is raining then there are clouds in the sky"
- According to the answer you gave to (i) (a) Write the three relevant statements.
- (ii) (a) Construct a truth table for $(p \rightarrow q) \vee (q \rightarrow p)$ and show that the formula is always true.
- (b) Compute the truth table for the NOR logic gate.
- (iii) (a) Draw symbols for the logic gates given below:
- NAND
 - XOR
 - AND
 - NOT
- (b) Write three functions regarding to XOR logic gate.
- (c) If A , B and C are inputs for logic gates and X is the output, draw a logic circuit for the logic statement $X=(A \text{ AND } B) \text{ AND } (C \text{ OR NOT } B)$.

(25 marks)

(Q6)

- (i) (a) An arithmetic progression is a sequence of the form $a, a+d, a+2d, \dots, a+(n-1)d$, where the initial term is " a " and the common difference is " d " and " a " and " d " are real numbers. Write the formula for the sum of the first n terms of an arithmetic progression using " a " and " d ".
- (b) In an arithmetic progression, the sum of the first 10 terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term.
- (ii) (a) A geometric progression is a sequence of the form $a, ar, ar^2, ar^3, \dots, ar^n$, where " a " the initial term and " r " the common ratio. Write the formula for the sum of the first n terms and hence deduce the sum to infinity.
- (b) A geometric progression has first term " a ", common ratio " r " and sum to infinity is 6. Another, geometric progression has first term $2a$, common ratio r^2 and sum to infinity is 7. Find the values of a and r .

- (iii) (a) Give the definitions of “greatest common divisor” (gcd) and “least common multiple” (lcm).
- (b) Use the definitions of part (iii) (a) and find:
- The gcd of 12 and 30
 - The lcm of 24 and 36
- } (All working steps must be given.)
- (iv) (a) Let “ b ” be an integer and “ m ” be a positive integer. What is denoted by “ $b \bmod m$ ”?
- (b) Find the value of $13 \bmod 5$.

(25 marks)

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