The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc/ B. Ed Degree Programme

Department	: Mathematics
Level	: 05
Name of the Examination	: Final Examination
Course Title and - Code	: Linear Programming- ADU5300
Academic Year	: 2020/21
Date	: 26-12-2021
Time	: 01.30 pm - 03.30 pm
Duration	: Two (02) hours

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **Six (06)** questions in **Four (04)** pages.
3. Answer any **Four (04)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labeled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

01. A manufacturing firm has three machines M_1, M_2 and M_3 and they produce two types of products A and B. The firm sells them at a profit of Rs 20 and Rs 10 on types A and B, per unit, respectively. Also the machines M_1, M_2 and M_3 have 150, 150 and 100 machine-minutes respectively. The following table provides the required processing time in minutes for each machine on each product:

Machine / Product	M_1	M_2	M_3
A	2	3	2
B	3	2	1

(a) By clearly defining *decision variables* and stating all the *constraints*, model this as a linear programming problem to *maximize* the total profit under the given conditions and solve it by means of *graphical* method.

Hence,

(i) identify the *constraint* that causes *degeneracy*.

(ii) determine which machine is NOT useful to the production in the firm.
Justify your answer.

(b) As per the new requirement in the market, the firm decides to produce at least 20 items from the product B. Resolve the problem by incorporating the new requirement using the same graph.

(c) To increase the total profit, the firm decides to purchase a new machine M_4 in addition to the old machines in operation. It is given that, M_4 has 100 machine-minutes and one unit of each type of product requires only one minute of processing time on M_4.
Give your recommendation on this decision of the firm with justifications.

02. (a) Use the *simplex* method to solve the following linear programming problem by representing the *unrestricted* variable (X_2) in terms of two non-negative variables $(X_4$ and $X_5)$:

$$\begin{aligned} &\textit{Maximize} && Z = X_1 + 2X_2 + X_3 \\ &\textit{Subject to} && 2X_1 - 2X_2 + X_3 \le 2, \quad \longleftarrow \quad (1^{st}\ \textit{constraint}) \\ & && 2X_1 + X_2 + X_3 \le 2, \quad \longleftarrow \quad (2^{nd}\ \textit{constraint}) \\ & && X_1 + X_3 \le 1, \quad \longleftarrow \quad (3^{rd}\ \textit{constraint}) \\ & && X_1 \ge 0,\ X_3 \ge 0 \text{ and } X_2 \textit{ unrestricted.} \end{aligned}$$

(b) Verify the result that you obtained in part (a) by means of replacing the *unrestricted* variable (X_2) by non-negative variables $(X_1$ and $X_3)$ using the 2nd *constraint* of the above problem.

03. A construction company has three subordinates and four tasks to be performed. Each subordinate can perform only one task in the allocated time. The inefficiency cost matrix for a particular subordinate assigned to a particular task is as follows:

Task / Subordinate	*I*	*II*	*III*	*IV*
1	2	6	4	4
2	3	4	4	3
3	2	5	6	5

(a) How the tasks should be assigned to subordinates so as to *minimize* the inefficiency cost?

Hence,

(i) identify the task which will go incomplete.

(ii) if the 2nd subordinate has the capacity to perform upto two tasks as all the tasks must be completed, what should the subordinates be assigned to do?

(b) Suppose the company decided only the 1st subordinate to be assigned upto two tasks and all the tasks must be completed on time, what should the subordinates be assigned to do?

(c) Based on the results that you obtained in the parts (a)(ii) and (b), recommend that which subordinate should be assigned upto two tasks to fulfill all the tasks with the minimum cost.

[Turn over

04. Briefly describe the role of artificial variables in Linear Programming.

(a) Use the graphical method to identify the *feasibility* and *boundedness* of the solution of the following problem:

$$\begin{aligned} &\textit{Maximize} && Z = X_1 + X_2 \\ &\textit{Subject to} && X_1 - X_2 \geq 1, \\ & && -X_1 + X_2 \geq 1, \\ & && X_1 \geq 0,\ X_2 \geq 0. \end{aligned}$$

(b) Apply the *big- M* method to solve the above problem.

Hence,

(i) determine whether the problem is *bounded* or not. Justify your answer.

(ii) verify the result that you obtained in part (a).

05. Let B and D be the matrices corresponding to the *basic* and *non-basic* variables in a standard form of a linear programming problem. If the coefficient matrices of the *basic* and *non-basic* variables are C_B and C_D respectively and b is the right hand side value matrix, state the *formulas* of the *index row column* and *objective function* in matrix from.

(a) Create the table for the *initial basic feasible solution* of the standard form of the following linear programming problem:

$$\begin{aligned} &\textit{Maximize} && Z = 2X_1 + X_2 \\ &\textit{Subject to} && X_1 + 2X_2 \leq 10, \\ & && X_1 + X_2 \leq 6, \\ & && X_1 - X_2 \leq 2, \\ & && X_1 \geq 0,\ X_2 \geq 0. \end{aligned}$$

(b) Perform the *revised simplex* method to show that the optimal solution to the above problem is $X_1 = 3$, $X_2 = 1$ and $Z^* = 7$.

06. Let X_{ij} be the number of units to be shipped from supplier i to customer j, where $i, j = 1, 2, 3$.

(a) Write down the *cost and rim* requirement table for the following transportation problem:

$$X_{11} + 4X_{12} + 3X_{13} \le 11,$$
$$3X_{21} + 2X_{22} + 5X_{23} \le 10,$$
$$2X_{31} + X_{32} + 2X_{33} \le 9,$$
$$X_{11} + 3X_{21} + 2X_{31} \le 7,$$
$$4X_{12} + 2X_{22} + X_{32} \le 12,$$
$$3X_{13} + 5X_{23} + 2X_{33} \le 11,$$
$$X_{ij} \ge 0 \text{ for all } i, j = 1, 2, 3.$$

(b) Find the optimal solution to the objective function $Z = 2X_{11} + 5X_{12} + 4X_{13} + 3X_{21} + X_{22} + 5X_{23} + 7X_{31} + 3X_{32} + 2X_{33}$, under the constraints in part (a), using *Minimum- Cost* method.

(c) Verify the result that you obtained in part (b) using Vogel's Approximation method.