(30)
00190

The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Numerical Methods – ADU5307**
Academic Year	**: 2020/21**
Date	**: 09.12.2021**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of (6) questions in (2) pages.
3. Answer any (4) questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

1. (a) Using Newton-Raphson method, show that the iteration formula for finding the p^{th} root of a is given by $x_{n+1} = \dfrac{(p-1)x_n^p + a}{px_n^{p-1}}$ where a is a real number and $n = 1, 2, 3, \ldots\ldots$

Hence find $\sqrt{5}$ correct to four decimal places.

(b) Show that for the equation $x^3 - x - 1 = 0$, there exists a root in the interval $(1, 2)$. Using the Simple Iteration method find the root correct to four decimal places, taking $x_0 = 1.3$.

2. (a) Prove that

(i) $\Delta = E - I$,

(ii) $\nabla = I - E^{-1}$,

(iii) $(I + \Delta)(I - \nabla) = I$ where Δ, ∇, E and I are the forward difference, backward difference, the shift and identity operators respectively.

(b) The following data provide the distance y in nautical miles of the visible horizon for the given heights x in feet above the earth surface. Find y when $x = 120$ and when $x = 380$.

Height: x	100	150	200	250	300	350	400
Distance: y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

3. State the Trapezoidal rule, Smpson's One -Third rule and Simpson's Three -Eighth rule.

Evaluate $\int_0^6 \dfrac{dx}{1+x^2}$ correct up to 4 decimal places, by dividing the interval $(0, 6)$ into 6 equal parts and using,

(a) the Trapezoidal rule,
(b) Simpson's One -Third rule,
(c) Simpson's Three -Eighth rule.

Find also its actual value correct up to 4 decimal places and compare it with the above results.

4. (a) Applying Taylor series method of fourth order for the differential equation $\frac{dy}{dx} = x^2 + y^2$ subject to the initial condition $y(0) = 1$, evaluate $y(0.1)$ and $y(0.2)$ to four decimal places.

(b) Using the Taylor series method of fourth order for the differential equation $\frac{d^2y}{dx^2} = y + x\frac{dy}{dx}$ subject to the initial conditions $y(0) = 1$ and $y'(0) = 0$, evaluate y(0.1)) correct to four decimal places.

5. (a) Using Picard's method, find the first-three successive approximations to solve $\frac{dy}{dx} = 2xy$ with the initial condition $y(0) = 1$.

(b) Applying Euler's method solve $\frac{dy}{dx} = y + x^2$ subject to the initial condition $y(0) = 1$ with step size $h = 0.1$, find the value of $x(0.5)$ for the initial value problem $\frac{dy}{dx} = \frac{y - x}{x + y}$.

6. (a) Applying Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ at $x = 0.2$, subject to the initial condition $y(0) = 1$.

(b) Applying Runge-Kutta method of fourth order, solve $\frac{d^2y}{dx^2} - y^3 = 0$ at $x = 0.1$, subject to the initial conditions $y(0) = 10$ and $y'(0) = 50$