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The Open University of Sri Lanka

Faculty of Natural Sciences

B.Sc/ B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title Code	**: Newtonian Mechanics II**
Course Title Code	**: ADU5303**
Academic Year	**: 2020/21**
Date	**: 24.03.2022**
Time	**: 1.30 p.m. To 3.30 p.m.**
Duration	**: Two Hours.**

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **(6)** questions in **(4)** pages.
3. Answer any **Four (4)** questions only. All questions carry equal marks.
4. Answer for each question should commence from a new page.
5. Draw fully labelled diagrams where necessary
6. Involvement in any activity that is considered as an exam offense will lead to punishment
7. Use blue or black ink to answer the questions.
8. Clearly state your index number in your answer script

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1. (a) In the usual notation, show that in Cylindrical polar coordinates, the velocity and acceleration of a particle are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + \dot{z}\underline{k}$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta + \ddot{z}\underline{k}$, respectively.

(b) A particle of mass m moves on the smooth inner surface of a hollow cylinder of radius b whose axis is vertical.

(i) Choosing Cylindrical polar coordinates suitably, show that

$$r^2\dot{\theta} = h \text{ where } h \text{ is a constant,}$$

$$\theta(t) = \frac{ht}{b^2} + \theta(0),$$

$$z(t) = z(0) + \dot{z}(0)t - g\frac{t^2}{2}.$$

(ii) If R is the reaction force of the cylinder wall on the particle then show that $R = \dfrac{mh^2}{b}$.

2. (a) In the usual notation, for a system with holonomic constraints derive Lagrange's equations $\dfrac{d}{dt}\left(\dfrac{\partial T}{\partial \dot{q}_j}\right) - \dfrac{\partial T}{\partial q_j} = Q_j$, $j = 1, 2, \ldots, n$ where T is the kinetic energy of the system.

(b) A long straight wire is rotating with constant angular velocity ω about a fixed point on the wire. A bead of mass m is sliding on the wire in a force free space.

(i) Show that the kinetic energy of the bead is given by
$T = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\omega^2.$

(ii) Using Lagrange's equations of motion, show that the equation of motion is given by $\ddot{r} = r\omega^2$.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

An object is projected vertically downward with speed v_0. Prove that after time t, the object is deflected east of the vertical by the amount $\omega v_0 \cos\lambda\, t^2 + \frac{1}{3}\omega g t^3 \cos\lambda$ where λ is the latitude of the point of projection and ω is the angular speed of the earth about its polar axis.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a conservative system with holonomic constraints are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = 0,\ j = 1, 2 \ldots, n.$

(You may assume the result in Question 2)

(b) A uniform rod OA of length $2a$ and mass m is smoothly pivoted at one end to a fixed point O. A bead of mass λm slides on the smooth rod and is connected to O by light elastic string of modulus nmg and natural length a. At time t, the rod makes an angle θ with the downward vertical OZ and the plane AOZ makes an angle ϕ with a fixed vertical plane. Show that the kinetic energy T of the system is given by

$$2T = \frac{4}{3}ma^2(\dot{\theta}^2 + \dot{\varphi}^2 \sin^2\theta) + \lambda m(\dot{x}^2 + x^2\dot{\theta}^2 + x^2\dot{\varphi}^2 \sin^2\theta)$$

where x is the stretched length of the string.

Also, write down an expression for the potential energy and use the result in 4(a), to obtain the equations of motion.

5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(b) If a rectangular parallelepiped with its edges $2a, 2a, 2b$ rotates about its center of gravity under no forces. Prove that, its angular velocity about one principal axis is constant and about the other axis it is periodic.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i} = \dot{q}_i, \quad \frac{\partial H}{\partial q_i} = -\dot{p}_i$.

The Hamiltonian of a dynamical system is given by

$H = q_1 p_1 - q_2 p_2 - a q_1^2 + b q_2^2$ where a and b are constants.

Obtain Hamilton's equations of motion and hence find p and q at time t.