The Open University of Sri Lanka

B.Sc/B.Ed. DEGREE, CONTINUING EDUCATION PROGRAMME

Final Examination - 2020/2021

Level 05 - Applied Mathematics

ADU5305/ADE5305– Statistical Inference

Duration: - Two Hours.

DATE: - 01-04-2022 **Time: - 9.30 a.m. – 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided.

Answer four questions only

1.

Suppose that the daily travel time to *ABC* school of a randomly selected student, follows a normal distribution. However, the mean travel time and variance travel time to the school is unknown. Suppose total number of students in the school is 800. Random sample of 20 students were selected and travel time to school was collected.

(i) What is the population of interest? Is the population finite? Justify your answer.

(ii) Derive moment estimators for the mean and the variance of daily travel time to school of a randomly selected student.

(iii) Daily travel time of 20 randomly students in minutes are given below. Using part (ii) estimate the mean and the variance of daily travel time to school of a randomly selected student.

94	70	43	28	62
11	27	66	34	46
66	79	66	74	20
98	58	60	73	69

(iv) Estimate the standard error of the estimated mean given by you in part (iii).

(v) Estimate the sample size required to estimate the mean daily travel time to school of a randomly selected student with an error bound of 5 minutes at 95% confidence.

(vi) Construct a 95% confidence interval for mean travel time to school of a randomly selected student in *ABC* school. Interpret the results.

2.

In a production process of an electrical product, the production manager is interested in the proportion θ of nonconforming electrical products produced. Suppose that the total production on a particular day is 20000. Random sample of 100 electrical products (drawn with replacement) were tested on this particular day by the production manager. Suppose that 9 of the electrical products in the sample were not according to the specifications.

(i) Construct a 95% confidence interval for θ.

(ii) Construct a 95% confidence interval for the total number of electrical products, produced on that day. Hence comment on the claim that " total number of nonconforming electrical products produced on that particular day is 200".

(iii) Using a suitable statistical test comment on the claim that "proportion θ of nonconforming electrical products produced on that particular day is greater than 0.1" at 5% level of significance.

(iv) Using Part (iii) test the validity of the claim that " total number of electrical products produced on that particular day exceeds 200" at 5% level of significance.

3

(a) Suppose that $\widehat{\theta}$ is an estimator for parameter θ. State whether the following statements are true or false. In each case justify your answer.

(i) $\widehat{\theta}$ is an unbiased estimator for parameter θ implies that $\widehat{\theta}$ is a precise estimator for parameter θ.

(ii) $Var(\widehat{\theta}) = \frac{\theta}{n}$ and $\widehat{\theta}$ is an unbiased estimator for parameter θ implies that $\widehat{\theta}$ is a consistent estimator for parameter θ.

(b) Let $X_1, X_2, X_3, \ldots\ldots X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{1-\theta} \quad ; \; 0 \leq x \leq 1-\theta ; \quad 0 < \theta < 1$$

(i) Find the mean of the above distribution.

(ii) Derive Maximum likelihood estimators for θ and for mean of the above distribution.

(iii) A random sample drawn from the above distribution is given in the following table.

0.127	0.196	0.279	0.576	0.46
0.054	0.695	0.322	0.48	0.4
0.657	0.188	0.003	0.062	0.198
0.612	0.295	0.125	0.675	0.577

Estimate θ and mean of the above distribution using maximum likelihood estimators derived in part(ii).

4.

(a) Briefly explain the following terms.

(i) Type I error and Type II error

(ii) Significance level and Power of the test

(b) Suppose that the distribution of weight of a tea bag (X) in grams, produced by *PQR* Company, follows a normal distribution. However, the mean weight and variance weight of randomly selected tea bag is unknown. Weights of 16 randomly selected products in grams are given bellow.

4.830	5.515	4.307	5.825	6.143
3.087	4.595	4.317	5.498	4.623
4.186	4.863	3.845	4.332	5.775
4.018	5.331	6.657	5.385	3.673

(i) Construct a 95% confidence interval for the variance weight of a randomly selected tea bag. Interpret your results.

(ii) Construct a 95% confidence interval for the mean weight of a randomly selected tea bag. Interpret your results.

(iii) Do the data provide evidence to justify the claim that "mean weight of a randomly selected tea bag is 5 grams". Justify your answer using part (ii).

5

(a) Briefly explain the following terms.

(i) Likelihood function

(ii) Mean square error of an estimator

(b) An investigation was conducted to compare the dust content in the flue gases of two types of solid – fuel boilers. 13 boilers of type *A* and 9 boilers of type *B* were used under identical fuelling and extraction conditions. Over a similar period, dust quantities, in grams, deposited in similar traps inserted in each of the twenty- two flues were collected. Assume that these samples come from normal populations. Sample means of dust contents of type *A* and type *B* are 63.83 grams and 52.89 grams respectively. Sample standard deviations of the dust contents of type *A* and type *B* are 10.63 grams and 9.00 grams respectively. From the past experience it is reasonable to assume that the variances are equal.

(i) Construct a 95% confidence interval for the differences of mean dust contents of type *A* and type *B*. Interpret the results.

(ii) Do the data provides evidence to justify the claim "mean dust contents of type A and type B are equal". Justify your answer using part (i).

6

(a) Briefly explain the following terms.

(i) Accuracy and Precision of an estimator

(ii) Sampling Distribution

(b) Final examination marks (out of 100) for Sinhala and History, of 16 randomly selected students are given below. Assume Sinhala and History marks follow normal distributions.

Student Registration Number	Sinhala Marks	History Marks
001A	48	60
002A	64	77
003A	37	37
004A	52	31
005A	58	48
006A	49	41
007A	41	38
008A	65	46
009A	26	22
010A	51	52
011A	69	80
012A	42	12
013A	67	64
014A	46	59
015A	17	30
016A	54	31

(i) Construct a 95% confidence interval for the mean difference of Sinhala mark and History mark of a randomly selected student. Interpret your results.

(ii) Using suitable statistical test, test the validity of the claim that "Expected Sinhala mark is greater than the expected History mark for a randomly selected student". Use 5% level of significance.

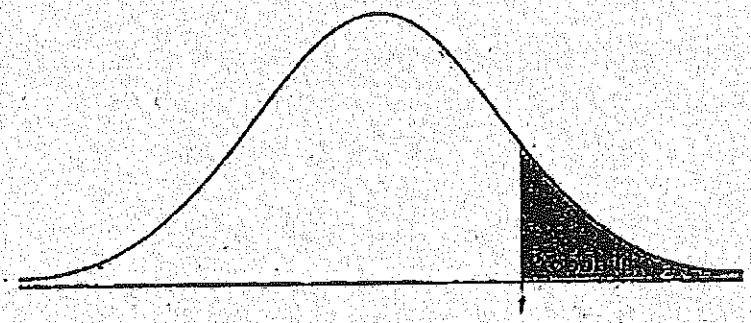


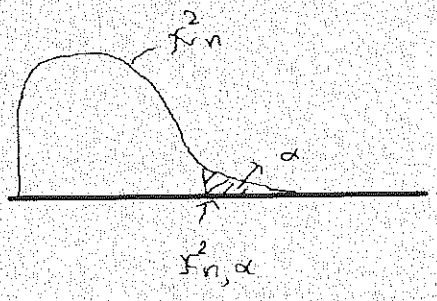
TABLE B: t-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850

Table of Standard Normal Probabilities

Let $Z \sim N(0,1)$. This table contains the probabilities $Pr(Z \geq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010



Chi-square Distribution Table

n ↓ d.f.	.995	.99	.975	.95	.9	.1	.05	.025	.01 → α
1	0.00	0.00	0.00	0.00	0.02	2.71	3.84	5.02	6.63
2	0.01	0.02	0.05	0.10	0.21	4.61	5.99	7.38	9.21
3	0.07	0.11	0.22	0.35	0.58	6.25	7.81	9.35	11.34
4	0.21	0.30	0.48	0.71	1.06	7.78	9.49	11.14	13.28
5	0.41	0.55	0.83	1.15	1.61	9.24	11.07	12.83	15.09
6	0.68	0.87	1.24	1.64	2.20	10.64	12.59	14.45	16.81
7	0.99	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98