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The Open University of Sri Lanka
Faculty of Natural Sciences
B.Sc. / B. Ed Degree Programme

Department	**: Mathematics**
Level	**: 05**
Name of the Examination	**: Final Examination**
Course Title and - Code	**: Fluid Mechanics – ADU5306**
Academic Year	**: 2020/2021**
Date	**: 26.03.2022**
Time	**: 09.30 a.m.-11.30 a.m.**
Duration	**: Two Hours.**

General Instructions

1. Read all instructions carefully before answering the questions.
2. This question paper consists of **six** and answer only **four** of them.
3. Each question carries 100 marks.
4. This paper consists of three pages.
5. Answer for each question should commence from a new page.
6. Draw fully labelled diagrams where necessary.
7. Involvement in any activity that is considered as an exam offense will lead to punishment.
8. Use blue or black ink to answer the questions.
9. Clearly state your index number in your answer script.

1.

(a) Briefly describe each of the following types of fluid motions:

i) Steady and Un-steady flows

ii) Compressible and Incompressible flows

iii) Rotational and Irrotational flows

(b) Find the acceleration components at a point (1,1,1) for the flow field with velocity components $(v_1,\ v_2,\ v_3)$, where

$$v_1 = 2x^2 + 3y;\ \ v_2 = -2xy + 3y^2 + 3zy;\ \ v_3 = -\left(\frac{3}{2}\right)z^2 + 2xz - 9y^2z.$$

(c) Suppose the stream function given by $\psi(x, y) = xy$ represents an irrotational flow. Find the velocity potential for the flow.

2.

(a) Derive the continuity equation of the form $\frac{D\rho}{Dt} + \rho\, div\left(\underline{q}\right) = 0$, for any arbitrary control volume of a moving fluid irrespective of its shape and size.

(b) Hence deduce the continuity equation, for an incompressible fluid in terms of Cartesian coordinates.

(c) Prove that the fluid $V = x^2y\,\underline{i} + y^2z\,\underline{j} - (2xyz + yz^2)\,\underline{k}$ is a steady incompressible fluid flow.

3.

(a) Given Euler's equation of motion $\underline{F} - \frac{1}{\rho}\, grad\, p = \frac{D\underline{q}}{Dt}$ for a perfect fluid. Show that it can be written in the form $\underline{F} - \frac{1}{\rho}\, grad\, p = \frac{\partial \underline{q}}{\partial t} + grad\left(\frac{q^2}{2}\right) - \underline{q} \times curl\, \underline{q}$.

(b) Using the result in part (a), derive Bernoulli's equation for irrotational motion of an inviscid homogeneous fluid of constant density.

(c) A horizontal nozzle reduces from $100\ mm$ bore diameter at inlet to $50\ mm$ at exit. It carries liquid of density $1000\ kg/m^3$ at a rate of $0.05\ m^3/s$. The pressure at the wide end is $500\ kPa$ (gauge). Calculate the pressure at the narrow end neglecting friction.

4. A two-dimensional source of strength $2m$ is placed at the point $A(a, 0)$ and a sink of strength m at the point $B(-a, 0)$. Write down the complex potential, and show that
   (a) There is a point of stagnation at $C(-3a, 0)$.
   (b) Points where velocity is parallel to the real axis lie on AB or on the circle of center C and radius $2\sqrt{2}\,a$.
   (c) The speed q at any point P in the z-plane is $\frac{m\,(\mathrm{PC})}{\mathrm{PA.\,PB}}$.

5. A stream of liquid has velocity V at infinity in the negative x-direction, the sphere $r = 2$, being a rigid boundary. Moreover, the velocity potential of the flow is given by $\phi = V\left(r + \frac{4}{r^2}\right)\cos\theta$.
   (a) Derive the components of velocity and hence obtain Stokes stream function.

$$\left(\text{Hint: } -\frac{\partial \phi}{\partial r} = q_r = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta} \text{ and } -\frac{1}{r}\frac{\partial \phi}{\partial \theta} = q_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}\right).$$

   (b) Find the equation of a streamline which is at a distance a from the axis, at infinity, and show that such a streamline meets the plane $\theta = \frac{\pi}{2}$, at a point where $r = b$ given by $\left(b^2 - \frac{8}{b}\right) = a^2$.

6. The complex potential of a fluid flow is given by $W(z) = U\left(z + \frac{4}{z}\right)$ where U is a positive constant.
   (a) Obtain the stream function and the velocity potential.
   (b) Find the complex velocity at any point in the form of $v_1 - iv_2$, where (v_1, v_2) are the components of velocity.
   (c) Find the stagnation point(s) of the flow.